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*PLANT HORMONES AND MINERAL NUTRITION**

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The relation of mineral nutrients to the production of growth hormone in plants is unknown. In order to determine the influence of certain materials upon the production of growth hormone in the shoot tips of plants, common sunflower, *Helianthus annuus* L. and Turkish tobacco, *Nicotiana Tabacum* L. var. Turkish, were grown in sand cultures. They were supplied with appropriate nutrient solutions (1) to alter the amount of nitrogen available, (2) to vary the proportion of NO_3 ions with respect to some other anions and to certain cations and (3) to alter the total concentration of the nutrient solutions in the range from $^1/_2$ to 3 atmospheres osmotic pressure. The effect of relative deficiency of some of the essential chemical elements was studied also in field-grown plants.

The nutrient solutions used were prepared in accordance with the directions of the authors cited and hormone concentration was determined by the Avena technique (Went).[1] In all hormone tests the agar plate used for diffusion was one-half of the standard plate described by Dolk and Thimann.[2] Hormone concentration is given in terms of curvature of Avena coleoptiles (in degrees). The method of handling and sampling the experimental plants is briefly described in the sections in which the data are reported.

Total Concentration of Nutrient Solutions.—In order to test the influence of the concentration of nutrient solution upon growth hormone in stem tips, Helianthus and Nicotiana were grown in sand to which was added four different concentrations of solution employed by Nightingale and Farnham.[3] Composition of the solutions is indicated in table 1. In this experiment the relative proportions of the constituent ions are kept constant, but the total concentration is varied to give pressures of approximately $^1/_2$, 1, 2 and 3 atmospheres.

Helianthus seedlings grown in these solutions were tested for growth hormone when twelve days old and at different intervals thereafter. For hormone determinations two excised shoot tips were "diffused" on an

TABLE 1

GROWTH HORMONE IN SHOOT TIPS IN RELATION TO TOTAL CONCENTRATION OF THE NUTRIENT SOLUTION

(Nightingale and Farnham[3] solutions)

Osmotic Concentration in Atmospheres	COMPOSITION OF NUTRIENT SOLUTIONS: Partial Volume-Molecular Concentration of Salts Employed			GROWTH HORMONE CONCENTRATION EXPRESSED AS AVERAGE AVENA CURVATURE, IN DEGREES	
	KH_2PO_4	$Ca(NO_3)_2$	$MgSO_4$	Helianthus Average of 7 Sets of Tests July 31–Sept. 11	Nicotiana Average of 6 Sets of Tests July 31–Sept. 11
1/2	0.0022	0.0045	0.0022	5.1	9.2
1	0.0045	0.0090	0.0045	6.2	8.0
2	0.0090	0.0180	0.0090	4.9	8.5
3	0.0135	0.0270	0.0135	4.7	10.2

agar plate, four plants being used for each concentration in each test. The Nicotiana plants were grown in soil until they had reached a height of about three inches. Then they were dug up, carefully washed and transplanted into the sand cultures. Growth hormone tests were made periodically, "diffusing" one excised shoot tip on an agar plate. Two plants grown in each concentration were used for each test.

The results are presented in table 1. The average Avena curvatures do not differ markedly throughout the period of the experiment. For Nicotiana, the range is from 8.0 to 10.2, and for Helianthus from 4.7 to 5.1. Evidently total nutrient salt concentration is without influence on hormone production in the stem tip (within the limits of this experiment).

Variation of the Nitrogen Supply.—The relationship of growth hormone concentration to the nitrogen nutrition of plants was tested with Helianthus and Nicotiana grown in modified Turner's[4] solutions in sand culture. Three different concentrations of nutrient solution (containing nine salts) as used by Turner were supplemented with one other to which no nitrogen was added (a six-salt solution). The total salt concentration in these four culture solutions varied from 0.525 to 2.025 g. per liter. Increasing concentration of nitrogen is indicated by designating the solutions as O, I, II and III. Solution O contains no nitrogen; solution I contains 0.025 g. each of KNO_3, $Ca(NO_3)_2$ and $Mg(NO_3)_2$; solution II has ten times as much nitrogen, and solution III has twenty times as much solution as I. The other constituents remain unchanged.

Helianthus plants were tested for growth hormone when ten days old and at intervals afterward. Nicotiana was tested upon four different occasions, starting when the plants were about five inches high. In the tests two shoots of Helianthus and one shoot of Nicotiana were used for "diffusion" on each agar plate, three or four plates being employed for each culture series in each set of tests.

The results show considerable difference in growth hormone production in stem tips of plants growing in the different solutions (Fig. 1 and Table 2).

The smallest amount of growth hormone and the least growth occurred in the culture which received no added nitrogen. Increased vigor and greater amounts of hormone were found in the plants receiving nitrogen,

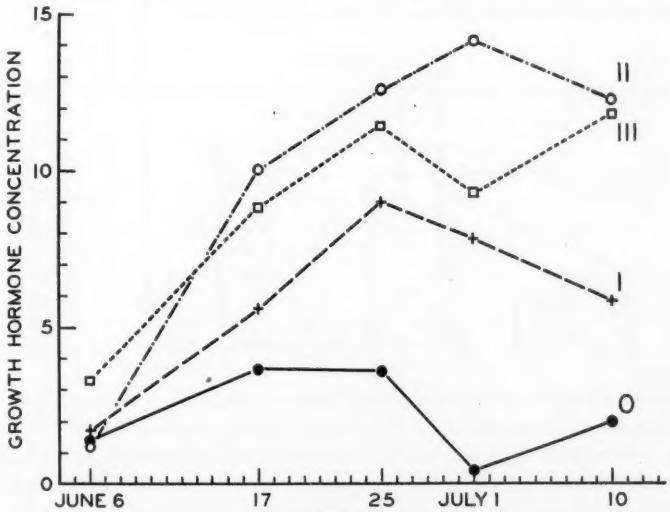

FIGURE 1

Relative growth hormone concentration in relation to varied nitrogen nutrition (Helianthus). Nitrogen is absent in solution O, and increases successively in solutions I–III. Hormone concentration is expressed as curvature of Avena coleoptiles, in degrees.

TABLE 2

GROWTH HORMONE IN SHOOT TIPS IN RELATION TO VARIED NITROGEN NUTRITION

(Turner's Solutions[4])

Culture Series	GROWTH HORMONE CONCENTRATION 0	I	II	III
Helianthus				
Average of 5 sets of tests, June 6–July 10	2.2	6.0	10.1	9.0
Nicotiana				
Average of 4 sets of tests, July 23–Aug. 21	1.6	10.9	12.2	11.0

in the order, Culture I, III, II. In this experiment the plants of group II exhibited the best growth and the highest hormone concentration. From the results of the experiments reported in the previous section it is clear that the differences in the osmotic pressures of the four solutions do not account for the differences in hormone concentration, which therefore may be attributed to nitrogen nutrition.

Varied Proportions of Nutrient Ions.—The quantity of growth hormone in plants in relation to varied nutrient ion proportions was studied in Helianthus grown in sand cultures. Solutions devised by Beckenbach, Wadleigh and Shive[5] were employed, permitting variation of the cations and anions in a definite manner as indicated in table 3.

TABLE 3

GROWTH HORMONE IN HELIANTHUS SHOOT TIPS IN RELATION TO VARIED NUTRIENT ION PROPORTIONS

(Beckenbach, Wadleigh and Shive[5])

NUMBER OF CULTURE SOLUTION	CATION PROPORTIONS			ANION PROPORTIONS			GROWTH HORMONE CONCENTRATION
	Ca	Mg	K	NO_3	SO_4	PO_4	
5	High	Low	Low	High	Low	Low	13.5
7	"	"	"	Low	High	"	7.4
8	"	"	"	"	Low	High	3.3
9	Low	"	High	"	"	"	5.6
11	"	"	"	High	"	Low	10.4
12	"	"	"	Low	High	"	1.5

The nutrient solutions, designated Nos. 5, 7, 8, 9, 11 and 12, each having an osmotic concentration of about 0.5 atmospheres, are made up with salts selected in appropriate amounts from the following list: KH_2PO_4, $Ca(NO_3)_2$, $MgSO_4$, $Ca(H_2PO_4)_2$, KNO_3, K_2SO_4 and $CaSO_4$. Detailed information concerning the composition of these solutions may be found in the original paper cited above.

Tests for growth hormone were made when the plants were twelve days old, and each week thereafter. The shoot tips of four plants growing in each kind of solution were selected for the tests, two shoots being "diffused" into each agar plate. The data for the two tests have been averaged and presented in figure 2 and table 3.

Considerable variation in growth hormone concentration is indicated for the different cultures. High proportions of nitrogen in the solution are associated with relatively large amounts of growth hormone and relatively vigorous growth, as indicated in figure 2. For comparison, dry weights of maize grown in the same culture solutions (data from Beckenbach, Wadleigh and Shive[5]) have here been included. Various proportions of other nutrient ions modify the response appreciably.

Deficiencies of Various Elements in Field-Grown Plants.—Supplementing observations on plants grown in nutrient sand cultures in the greenhouse, tests for growth hormone were made upon Nicotiana growing in experimental field plots at Marlboro, Maryland. This work was made possible through the kindness of Dr. J. E. McMurtrey, Jr., in charge of the U. S. Department of Agriculture fertilizer investigations, and the coöperation of Dr. E. S. Johnston of the Division of Radiation and Organisms, Smith-

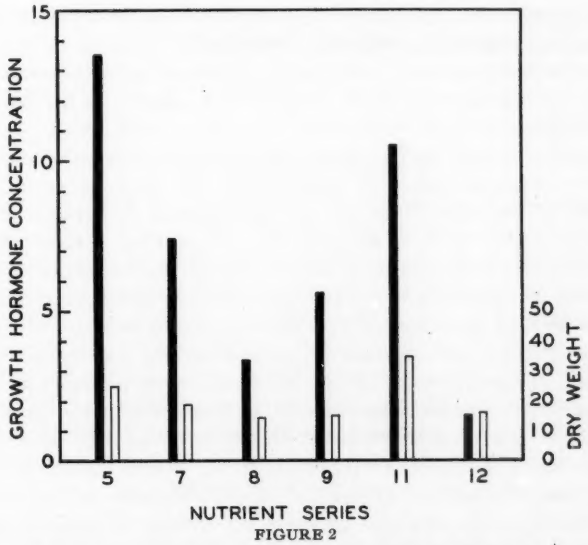

FIGURE 2

Relative growth hormone concentration in Helianthus in relation to varied proportions of certain nutrient ions (heavy black columns). The numbers of the nutrient series are those used by Beckenbach, Wadleigh and Shive.[5] Hormone concentration is expressed as curvature of Avena coleoptiles, in degrees. Dry weights, in grams, are given for other plants grown in corresponding solutions (light columns).

sonian Institution. Duplicate tests were made on plants showing pronounced deficiency symptoms by "diffusing" each excised shoot tip upon an agar plate in the field. Then the blocks were placed on ice for transporting to the Smithsonian laboratory in Washington, where the Avena coleoptile tests were made. The results are shown in table 4.

TABLE 4

GROWTH HORMONE IN SHOOTS OF NICOTIANA. FIELD GROWN PLANTS

	CONTROL		RELATIVE DEFICIENCIES									
	In-florescence	Vegetative Shoot	−Mg	−Mg	−Ca	−Ca	−K	−B Infl.	−B Veg.	−P	−N	"FRENCHED" NORMAL
Growth Hormone Concentration	19.8	23.1	25.0	20.6	21.2	23.6	21.0	17.6	21.0	22.3	13.4	23.2

Though deficiency symptoms were pronounced in the plants selected for tests, the growth hormone present differed but little in control and deficient plants. Presumably the degree of deficiency manifested in the

external appearance of the shoots was not sufficient to influence appreciably the formation of growth promoting substances.

Of considerable interest is the finding of reduced growth hormone concentration in the shoot tips of "Frenched"[6] tobacco as compared with normal plants.

The results of field studies when compared with those obtained with laboratory cultures (nitrogen deficiencies, for example) indicate the superiority of the latter, in which more pronounced deficiencies are easily obtainable. This is of importance, for in making determinations of growth hormone it is necessary to test regions such as shoot tips where appreciable concentrations of hormone occur. Available materials in the organism tend to be mobilized in a growing region such as the shoot tip, and growth there may continue in spite of certain unfavorable nutrient conditions. Hence, in most instances tests of the tip probably do not give a true indication of malnutrition unless the cultural deficiencies are extreme.

Summary.—The growth hormone present in shoot tips of Helianthus and Nicotiana was determined for plants grown in sand culture under varying conditions of mineral nutrition: (1) Growth hormone is scarcely detectable in the shoot tips of plants grown in the absence of supplied nitrogen; (2) when the relative proportions of certain anions and cations are varied, growth hormone is most abundant in the shoot tips of plants grown with high concentrations of nitrogen; (3) growth hormone production in shoot tips is unaffected by relative deficiencies of the common elements in field experimental fertilizer plots; (4) variations in osmotic pressure of the nutrient solutions in which the plants are grown does not affect appreciably the growth hormone production in shoot tips; (5) there is no evidence in these experiments to show whether the concentration of growth hormone in shoot tips of plants is the cause or effect of different growth rates brought about by varying the mineral nutrient supply.

* This contribution is one of a series made possible by the Rockefeller Foundation, in coöperation with Connecticut College.

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[6] Wolf, F. A., Duke Univ. Press, Durham, N. C. (1935).

A SIMPLIFIED EXPLANATION OF BELLAMY'S[1] EXPERIMENTS CONCERNING SEX DETERMINATION IN TROPICAL FISHES

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Individual animals are distinguished as male or female according as they produce sperm or eggs at reproduction. If an individual produces both eggs and sperm, it is known as a hermaphrodite. Production of eggs and production of sperm take place at different levels of metabolic activity, egg production involving greater development of reserve food materials for the nourishment of the future embryo, whereas the sperm includes scarcely more than the essential nuclear constituents of a gamete.

In the early years of the present century it was definitely established that the determination of the sex of individual offspring is a genetic phenomenon, involving the production by one parent, either of two kinds of eggs (potentially male and female, respectively), or of two kinds of sperm (male determining and female determining).

The first case experimentally demonstrated was that of the squash bug, *Anasa tristis*, in which two kinds of sperm are produced by the male parent (male determining and female determining, respectively), the female parent producing only a single type of egg, which after maturation contains 11 chromosomes. The sperm cells of this species differ in chromosome number, male-determining sperm containing 10 chromosomes, female-determining sperm containing 11 chromosomes. The egg which is fertilized by an 11-chromosome sperm becomes a female (with 11 + 11 chromosomes in the zygote); the egg which is fertilized by a 10-chromosome sperm becomes a male (with 11 + 10 chromosomes in the zygote). In this case we may speak of the male as the heterogametic sex (since it produces two unlike types of gametes) the female being homogametic (producing only one type). The odd chromosome of the male (the one without a mate) had been called the *X*-chromosome by cytologists. It now became recognized as a sex-determinant. The formula of the sexes accordingly was XX = female, XO = male. Later Morgan showed that in Drosophila the *X*-chromosome is the vehicle of sex-linked inheritance. He supposed that in the male Drosophila (as in the squash bug) the sex-chromosome would have no synaptic mate. But subsequent cytological study showed that it had a mate, which was designated as *Y*, but this apparently carried no genes. I pointed out the fact that if it did come to carry genes, these would be capable of transmission directly from father to son, unlike genes borne in an *X*-chromosome. This form of transmission of characters was some years later shown to exist in fishes by Schmidt and independently by Aide.

Also it was found that the *Y*-chromosome of Drosophila in reality may carry genes, contrary to what had previously been supposed.

The type of sex determination found in the squash bug and Drosophila occurs also in man and mammals generally, but in birds among the vertebrates and in moths among insects, a fundamentally different type of sex determination occurs. Here the female is the heterogametic sex and sex-linked characters are borne in a chromosome having male determining influence, which came to be called *Z*, its synaptic mate with female determining influence being called *W*. The formula of the sexes is then ZW = female, ZZ = male.

I have always regarded the difference in terminology used in describing the two types of sex determination as unfortunate and apt to be misleading and so suggested some years ago that it would simplify matters if we retained the original (*X*-*Y*) terminology in both cases calling a chromosome which has a female influence in sex determination always *X*, and one which has a male influence always *Y*. The constitution of the sexes would then be:

	Female	Male
In Drosophila and man	XX	XY
In birds and moths	XY	YY

The essential difference between the two systems would seem to consist in a difference in the relative potency of *X* and *Y*, *Y* being dominant in the Drosophila system and *X* in the poultry system. How such a change could have come about, we can only speculate. We find experimental evidence in the work of Bellamy that the Drosophila system is the original one and the poultry system the derived one. This work of Bellamy and others on sex determination and sex linked inheritance in tropical fishes is illuminating because here related species capable of being crossed have contrary types of sex determination. Bellamy has made reciprocal crosses between two species of Platypoecilus, *P. variatus* which has the Drosophila type of sex determination, and *P. maculatus* which has the poultry type.

Employing the simplified terminology which I am suggesting, these crosses may be expressed thus:

	Maculatus ♂	× Variatus ♀
Cross 1.	$Y_{\dot{p}}Y_{\dot{p}}$	XX

The maculatus *Y* chromosome bears in this cross a dominant sex-linked color gene (*pulchra*, $\dot{p}$). Since both parents are homogametic, the offspring (118 in number) are all alike $XY_{\dot{p}}$ in formula, all *pulchra* in color and male as to sex. This result indicates that when the two types of sex-deter-

mination come into conflict, Y (the male influence) is dominant over X (the female influence) as in Drosophila, suggesting that to be the more primitive type.

The reciprocal cross may be expressed thus:

Cross 2. Variatus ♂ × Maculatus ♀

$X\mathbf{Y}_{st}$ $\mathbf{X}Y_+$

Both parents are in this cross heterogametic, **Y** being dominant in the Variatus (male) parent and carrying a gene for striping (st), **X** being dominant in Maculatus, the female parent. The Maculatus parent used in this cross carried wild color (+), not *pulchra* in its Y chromosome. Four sorts of combination are possible, as follows:

Variatus gametes			X	$\mathbf{Y}_{st}$
Maculatus gametes			Y_+	**X**
Zygotes	XY_+	$Y_+\mathbf{Y}_{st}$	$\mathbf{X}X$	$\mathbf{XY}_{st}$
	13 ♂♂	10 ♂♂	2 ♀	1 intersex
	wild type	striped		?

The striping of 10 males indicated that they were $Y_+\mathbf{Y}_{st}$, the non-striping of 13 other males indicated that they were XY_+ (weak Y being dominant over weak X), as in the reciprocal cross. Two females would seem to answer to the category $\mathbf{X}X$. One of these females mated to a brother produced an intersex, which was probably **XY** in formula.

Hybrid F_1 males produced by Cross 1 (XY_p in formula) were backcrossed with females of both parent species with results which can be expressed thus:

Backcross 1. F_1 ♂ × Maculatus ♀

XY_p $\mathbf{X}Y_+$

F_1 gametes	X	Y_p		
Maculatus gametes	**X**	Y_+		
Zygotes	$\mathbf{X}X$	$\mathbf{X}Y_p$	XY_+	Y_pY_+
	10 ♀♀ wild,	49 ♀♀ pulchra,	9 ♂♂ wild,	33 ♂♂ pulchra

Backcross 2. F_1 ♂ × Variatus ♀

XY_p XX

F_1 gametes	X	Y_p	
Variatus gametes all	X		
Zygotes	XX	XY_p	
	69 ♀♀ wild type	49 ♂♂ pulchra	

Although Bellamy does not use the simplified terminology which I am suggesting, his interpretation is substantially the same as I have given. The essential points are that in *P. maculatus* sex-linked characters are borne in a *Y* (male determining chromosome) whereas in *P. variatus* they are borne in an *X* (female determining chromosome). When the two systems are brought into competition, *Y* is dominant over *X* as in Drosophila, this being thus indicated as the more primitive system. It is not clear what is the result when the strong *X* of one system is combined with the strong *Y* of the other system; possibly it results in the production of an intersex.

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THE KINETIC BASIS OF CRYSTAL POLYMORPHISM

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1. Introduction.—The existence of a substance in several crystalline modifications and the inversions of these from one form to another are phenomena usually thought of from a very generalized thermodynamic viewpoint. Tenable specific reasons why a substance should exist in more than one modification and why these should transform to one another under various changes of conditions do not appear to have been offered. In the present paper, an attempt will be made to give such reasons by suggesting a certain dynamic-structural viewpoint. For simplicity, change of phase with pressure will be neglected and attention will be focused on the change of phase with temperature.

The necessity for some degree of periodic and symmetrical arrangement in crystals requires that there be discernible within them some sorts of coördination groups of atoms. These coördination groups are well known,[1] and Pauling has formulated rules[2] for the prediction of structures of ionic crystals based upon coördination. The rules do not, however, give a unique solution of the crystal structure for any given compound, for there are ordinarily a number of alternative ways of linking groups together, each of which leads to a different structure. Such alternatives are, of course, possible polymorphous modifications of the same compound. It is believed that a development of the theory presented in this paper may afford a key to the correct selection of the appropriate crystal structure from the possible alternatives.

Cluster Vibrations in Crystals.—The temperature of a crystal implies an energy content in the form of motion of its fundamental atomic units. Since the energy is communicated to an atom mechanically by its neighbors, it can be said that the motions of the atoms are not entirely at random, but are synchronized, in a general way, with those of their neighbors. That is, heat motion is a wave motion among the atoms of the crystal structure. In the opinion of the writer, the secret of polymorphism resides in the form of the wave motion.

It is more convenient to examine the form of wave motion by fixing attention on the motion in the immediate vicinity of one of the major repetitive units in the crystal structure. Such a unit may be termed a *cluster*, which may be either a molecule, a molecular group of atoms or a coördination group of atoms. The three degrees of freedom of each of the n atoms of a cluster may be combined in various ways to give characteristic cluster vibration modes. Six degrees of freedom may be ascribed to translations and rotations of the clusters as bodily units. These may be regarded as "outer" vibration modes. The remaining $3n - 6$ degrees of freedom correspond with kinds of internal cluster distortions, etc., i.e., to "inner" vibration modes.[3] If the cluster is symmetrical (for example, the tetrahedral SiO_4 cluster), then many of these modes are degenerate, or turn out to be geometrically identical with one another except that they have different orientations; the number of distinctly different types of vibration states may thereby be greatly reduced. Thus, the tetrahedral SiO_4 cluster has $3 \times 5 - 6 = 9$ inner vibration modes, but because of the symmetry of the cluster, only 4 of these are geometrically distinct.

The response of a crystal to temperature amounts to the acquisition of one or more vibration modes by its clusters, for the motions of an array of atoms can be factored into a combination of one or more of its cluster vibration modes. Furthermore, it is now a familiar matter from Raman spectroscopy that the different modes are excited with different energy inputs. (This can be developed in the present connection from a consideration of bond-breaking tendency, or vapor pressure, for a given energy content of the several vibration modes.) One may ordinarily expect, then, that in simple cases, at least, the clusters of a crystalline material will tend to be excited to different possible cluster vibration modes with different levels of kinetic energy input, i.e., with different temperatures.

The excitation of outer or inner modes will depend primarily on the relative strength of inter-cluster versus intra-cluster bindings. The smaller this ratio, the more easily will the cluster behave as a rigid unit. In the case of molecule clusters and molecular group clusters, the weakness of the intermolecular forces permits very easy excitation of the outer modes, and as a consequence, the rotational motions of molecules in crystals[4,5,6] are quite common as a form of heat motion. In linked coördination struc-

tures, on the other hand, one would expect heat motion almost exclusively in the form of inner vibration modes.

The various vibration modes, in general, have different symmetries. In other words, the dynamic symmetries of a cluster are not necessarily identical with its static symmetry, but may be either higher or lower. The rotational outer modes ordinarily tend to increase the cluster symmetry to radial symmetry about the axis of rotation if the rotation is complete. The inner modes on the other hand, either have the symmetry of the ideal static cluster or a lower symmetry. From this it follows that if a crystalline material responds to a certain temperature with a certain cluster vibration mode, the clusters have a definite dynamic symmetry which requires a cluster linking (i.e., a crystal structure) in which the cluster occupies this symmetry in the space group of the structure. Another way of looking at this is that the heat motion wave must have the appropriate symmetry of the dynamic state of the cluster required at that temperature, and that a certain crystal structure gives the appropriate wave symmetry.

The gist of this theory of polymorphism may be summed up as follows: *For a given temperature, there is a preferred cluster vibration mode (or mode combination) leading to least vapor pressure; this vibration mode requires the cluster to have a certain dynamic symmetry. Stable crystals form at this given temperature by the building up of a crystal structure such that this preferred symmetry is provided by the structure for the cluster.*

It would be of the greatest interest to examine the known one-component systems in the light of the theory of polymorphism just suggested, with a view to testing it. In order to make such studies, not only must the thermodynamic relations among all the stable modifications be completely known, but the crystal structures of the several modifications as well as the energy relations between the several vibration modes of the cluster types must also have been worked out. These data are available for very few systems. In forthcoming papers, some of these will be considered.

Another interesting test of the theory suggests itself: A compound ought to be continually emitting possible electromagnetic radiation due to thermal vibrations. The radiation spectra ought to be different for the several modifications and for each modification the spectrum ought to correspond with the frequency or frequencies of the vibration mode or modes of the cluster in that modification. The emitted radiation for each modification ought to be the same as certain of the lines of the infra-red type absorption spectrum for that modification, and also correspond, where permitted, with certain of the lines of the Raman spectrum for that modification. In at least some instances, such as when the modifications are capable of existing at high temperatures, the emitted radiation ought to become strong enough to permit detection, measurement of frequencies

and correlation with infra-red absorption and Raman frequencies for the several modifications.

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THE GENERAL RÔLE OF COMPOSITION IN POLYMORPHISM

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The variables ordinarily considered to fix the state of a system are temperature and pressure. These are adequate provided that the system is truly a one component system. In another paper,[1] the structures assumed by a definite chemical compound in response to temperature changes are discussed, the pressure variable being kept constant purely for the sake of fixing attention on the thesis. We now require to know in what manner changes in the chemical composition of the substance may modify the structural types assumed. This involves the introduction of a new variable, composition, which gives to the system an additional degree of freedom. In a discussion given in a later paper, it will be demonstrated that in many cases where a system has been assumed to be a one component system, it has actually not been so, a procedure which has probably led to much confusion.

In a large way, the problem here is one of accounting for temperature-composition diagrams, for which the simplified form is the familiar equilibrium diagram, which deals only with stable phases. In the main, attention will be particularly directed to the region in the immediate vicinity of the original pure compound.

The mechanism of a change of composition in a given phase is solid solution. Three simple types of solid solution are possible (1) addition or interstitial solid solution, (2) proxy solution and (3) omission solution. These are discussed elsewhere.[2,3] The problem now resolves itself into an inquiry into what alteration each of the above conditions imposes upon the configuration and dynamic picture already presented[1] for polymorphism.

The most obvious effect of solid solution on the modification of the phase regions is found in the interstitial solid solution type. Interstitial solid solution can only occur when interstices of a size commensurate with the additional atom to be accommodated are available in a structure. Low temperature, compact forms therefore have a much more limited capacity for this type of solid solution than the open high temperature forms.[4] If a high temperature form of a compound can accommodate the excess atoms and the low temperature form cannot, then it follows that the stability region of the low temperature form may be eliminated entirely in favor of that of the high temperature form. In less extreme cases of differential solubility, then, it is still likely that the temperature range of the high temperature form will be extended downward.

Some examples of this sort have already been discussed for the silica system,[4] in which system it was pointed out that the compact quartz type is eliminated and the stability regions of the more open forms, tridymite and cristobalite, are extended downward because of the solution of additional large atoms.

The direction of the migration of a transformation point with impurity content is in harmony with Le Chatalier's Rule put in the following specific form: If an impurity is to be housed, then the modification best adapted to housing it will tend to form. The effect of foreign atoms in substitution solid solution on the migration of transformation points is less obvious, but probably this same rule may be invoked, at least in simple cases. Thus, other things being equal, it might be expected that if the substituting atom or group of atoms is of larger size than the one for which it is substituting, then the inversion point between an open and compact or collapsed structure should be lowered, or if it is smaller, it ought to be raised. This latter alternative seems to have certain distinct physical limitations. Volume considerations are not the only ones which may be treated by this rule; any tendency on the part of the new atom wherein it departs from the tendency of the replaced atom ought to be relieved by the extension of the stability region of the particular structure in such a direction as to relieve this departure. For example, if the proxy atom has a greater bonding tendency, then it ought to extend the range of the structure of greater coördination at the expense of the structure of lesser coördination. Ordinarily, but not necessarily, this would be the low temperature structure.

If the proxy atom has less bonding tendency, the range of the lower coordination structure ought to be extended.

Turning, now, to the omission type of solid solution, one notes that one of its necessary characteristics is the occurrence of the void spaces of omission. Assuming that the high temperature form is the open one, then, at the transformation temperature where this structure is incipiently unstable, a change of structure to introduce further void spaces might be expected to lead to a collapsing or compacting tendency which would extend the range of the compact or collapsed forms. In such simple cases, then, the transformation temperature ought to be raised by the deviation of the compound from the ideal formula.

In the foregoing discussion, space requirements have received the major consideration. The writer by no means regards these considerations as the only ones to be taken account of, nor even that they are very generally applicable ones, in the prediction of the migration of a transformation point with composition. The space requirements are, however, simple and obvious ones to consider in a preliminary discussion, and they do have the advantage of being able to explain some simple cases of the rôle of composition. It is certain that a change in composition must affect the dynamics of the vibrations[1] by disturbing their symmetry and magnitude, but a specific discussion of this is not required in the present paper.

An important conclusion may be drawn from the recognition of the fact that the transformation temperature may be shifted with composition: If a crystalline phase occurs or is prepared, and this phase type is known to be metastable for the pure compound, it cannot be concluded that the phase is actually metastable unless it can be demonstrated that the compound is actually pure. If this is not the case, i.e., if it contains even slight traces of impurity, the transformation temperature may have been so shifted by the presence of the impurity that the phase type metastable for the pure compound is actually stable for the impure compound. The usual methods of examining a phase are ones based upon physical properties which are not sensitive to small changes in chemical composition, so that unless very careful chemical analysis is also made, the oversight of a minor composition change is easy.

In recent years, the significance of impurities in crystals has begun to be recognized and its bearing upon the entire subject at hand will be discussed in another place.

In an early paper,[4] the idea of a *stranded phase* was introduced, and in a later article[5] it is suggested that the idea can be extended to phases stranded by an internal change of composition. To illustrate the significance of such a *compositionally stranded phase*, consider the following hypothetical situation:

A compound AB exists which may occur in two modifications, I and II. Under normal conditions, modification I is stable, modification II is unstable and tends to transform into I but is actually prevented from doing this by reason of a sluggish transformation which connects the two forms. Suppose that this compound, AB, is now precipitated in an impure form[3] $A\left|\begin{matrix}B_1-n\\C_n\end{matrix}\right|$ and the impurity content affects a shift in the transformation point of the substance such that modification II is actually formed under the new normal conditions. Modification II is accordingly the precipitated phase. Suppose, further, that this crystal of composition $A\left|\begin{matrix}B_1-n\\C_n\end{matrix}\right|$ and of modification II is allowed to remain in an environment which effects a removal of the impurity C in exchange for B, thus generating a pure compound of ideal composition AB. The interchange of C for B takes place in such a way that there is continuity of the original phase II structural configuration. When the pure composition condition, AB, is finally attained, in this interchange process, the phase II is no longer stable, but is a configuration condition which has been inherited from a different composition. The new phase II of AB is consequently *compositionally stranded.*

In conclusion, it should be pointed out that Barth's[6] definition of polymorphism, namely:

"Polymorphism includes every possible difference encountered in the crystalline lattice of a substance of constant chemical composition, excepting homogeneous deformations,"

is much too narrow. If the general form of this definition is acceptable as a definition of polymorphism, then it ought to be broadened to read:

"Polymorphism includes every possible difference encountered in the crystalline structure of a substance, excepting homogeneous deformations."

In the revised definition, the significance of the word "substance" is not the same as the narrow Daltonian significance of the word "compound," but has the broader significance recognized in the fields of mineralogy and metallography. The common factors influencing polymorphic features are not only temperature and pressure, but also composition.

The writer is indebted to Mr. M. C. Bloom of this laboratory for discussion of many phases of the problem of the rôle of composition in polymorphism.

[1] M. J. Buerger, "The Kinetic Basis of Crystal Polymorphism," *Proc. Nat. Acad. Sci.*, **22**, 682–685 (1936).

[2] M. J. Buerger, "The Pyrite-Marcasite Relation," *Amer. Min.*, **19**, 53–58 (1934).

[3] M. J. Buerger, "The Temperature-Structure-Composition Behavior of Certain Crystals," *Proc. Nat. Acad. Sci.*, **20**, 444–453 (1934).

[4] M. J. Buerger, "The Silica Framework Crystals and Their Stability Fields," *Zeit. Krist.*, **90** 186–192 (1935).

[5] M. J. Buerger, "Polymorphic Transformations," in preparation.

[6] Tom. F. W. Barth, "Polymorphic Phenomena and Crystal Structure," *Amer. Jour. Sci.*, **27**, 273 (1934).

NOTES ON NORTH AMERICAN LEPTODEIRA

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A study of the herpetological fauna of Nicaragua, Costa Rica aud Panamá, begun under a John Simon Guggenheim Memorial Fellowship in 1928, was further advanced by a grant-in-aid from the National Research Council in 1933. At that time it was found necessary to examine a number of Leptodeira from other countries in order to settle the status of those in lower Central America. Thus there has accumulated a mass of data which I have been urged to summarize for the benefit of other workers.

The genus *Leptodeira* (including *Hypsiglena*) consists of: Colubrine snakes of moderate to small dimensions; body cylindrical or laterally compressed; head somewhat to very distinct from neck; eye with vertical pupil, small to rather large; vertebrae without posterior hypapophyses; hemipenis single, with single sulcus, proximal hooks and distal calyces, the areas so furnished about equal, calyculate area with free proximal edge ("capitate"); maxillary teeth 10–16, subequal or increasing posteriorly, followed after a gap by two enlarged fangs, which may or may not be grooved; head scalation normal; dorsal scales in 17–25 rows, smooth, with two apical pits, vertebral and paravertebrals sometimes enlarged; ventrals normal; anal nearly always double; subcaudals paired; United States to Argentina.

Save for the genus *Trimorphodon*, which occurs with *Leptodeira* from the southern United States to Costa Rica, there are no close American allies. *Trimorphodon* have enlarged anterior teeth, more than one loreal and are larger snakes. I regard them as derivatives of *Leptodeira*.

The African genus *Crotaphopeltis* is stated to be a close ally of *Leptodeira*, differing in having a single anal.

Leptodeiras without grooved fangs are usually considered a different genus (*Hypsiglena*), and by some authors placed in a different subfamily. "Hypsiglena" consists of the spotted *torquata*, closely similar to *Leptodeira pacifica;* the broad banded *latifasciata*, closely similar to *Leptodeira mystacina* and to *L. nigrofasciata*, and specifically identical with *Leptodeira guilleni;* the narrow banded *discolor*, rather similar to *L. maculata*.

Thus I do not believe that the grooveless snakes form a natural group. Furthermore there is no difference save presence or absence of grooves between *Comastes ornata* Bocourt described from Panamá, and Panamanian specimens of *Leptodeira annulata*, nor between the Mexican *Hypsiglena latifasciata* and *Leptodeira guilleni*, so that I scarcely consider grooving of specific value in this group. The three so-called Hypsiglenas are far more different from each other than they are from various Leptodeiras. In these cases I consider the grooving of the maxillary teeth to have been lost.

In South America *annulata* is the only form in the southern and eastern regions. In Panamá *annulata* occurs with a more abundant (ratio of 1 to 6) form *rhombifera*, which enters South America, but which is replaced in the area from Peru to Venezuela by other species or races which occur with *annulata*. These forms I have not yet been able to work out. A third species, *bakeri*, of a quite different group, allied to *nigrofasciata* and *mystacina*, occurs on the island of Aruba.

North of Panamá the number of forms in any given area increases to a maximum on the west coast of Mexico, whence eight different forms have been examined (five occur in the adjacent states of Colima and Sinaloa). Here they are accompanied by *Trimorphodon*. On the Mexican plateau and north into the United States the number diminishes, *Trimorphodon* just crosses the border and the only form of *Leptodeira* widespread in the north is *ochrorhyncha*.

I shall not here detail the institutions whose lower Central American material has been examined. The list is long and will appear elsewhere. I have seen the entire collection of the Museum of Comparative Zoölogy at Harvard, and of the Academy of Natural Sciences in Philadelphia. I have examined all the North American material in the United States National Museum and in the American Museum of Natural History. I have seen a number of specimens from the Museum of Zoölogy of the University of Michigan and been provided with data on others. I wish to express my gratitude to these institutions and to the foundations which have enabled me to examine the material. The North American material of this group which I have been able to see amounts to 704 specimens from lower Central America, 250 from further north and 3 with no data; 957 in all. Of one form, *rhombifera*, I have seen 606 specimens. I am compelled to regard as valid three forms of which I have seen no examples. In the field I have met with the common forms *annulata*, *rhombifera* and *maculata*.

An 880 mm. long specimen of *L. rhombifera*, now in the Museo Nacional de Costa Rica, from Alajuela, C. R., bit a boy on the hand and caused marked, but not very serious, symptoms of poisoning.

Leptodeira rhombifera is the form which, in my opinion, is the most primitive. It is one of the largest species, has as many rows of dorsal

scales as any and more than most, is a spotted form, is certainly closer to *Trimorphodon* than any other, is very widely distributed and occupies a central position in any scheme of relationships. I should regard the banded forms, the smaller spotted forms and the arboreal forms as necessarily connected, one with another, by way of some such form as *rhombifera*.

KEY TO NORTH AMERICAN LEPTODEIRA

A. Vertebral (or vertebral and paravertebrals) scale row enlarged (no dark nape stripe; dorsal and lateral spots; body compressed; scales 19–23, usually 21, in South America occasionally 17; ventrals 186–199; caudals 81–84 ♀, 94–97 ♂; fangs grooved; Nicaragua, Costa Rica, Panamá, into South America)..... *annulata annulata.*

AA. Vertebral scale row not enlarged.

B. Spotted snakes, the dorsal spots or rhombs not reaching scale row 1; lateral spots usually well marked.

C. A dark nape stripe; caudals more than 60; snakes fairly large; scales 21–25.

D. Ventrals 193–211 (dorsal spots usually small; body compressed; scales 21–23, usually 23; caudals 75–102; fangs grooved; Vera Cruz and Colima to Honduras)..... *annulata polysticta.*

DD. Ventrals 158–178 (dorsal spots large; body cylindrical; scales 21–25; caudals 64–84; fangs grooved; Sinaloa and Vera Cruz south through Central America—except for Yucatan, Peten and British Honduras—and into South America)..... *rhombifera.*

CC. No dark nape stripe or caudals not more than 55; small snakes; scales 19–21; body cylindrical.

D. Caudals 61–70; fangs grooved (a black collar; tiny spots in numerous rows; scales 19; ventrals 152–164; Sinaloa and Jalisco).. *pacifica.*

DD. Caudals 43–55; fangs not grooved.

E. A black collar with forward extensions to nape and eye; scales 21.

F. Ventrals 178–186; Lower California..... *torquata venusta.*

FF. Ventrals 167–175; Lower California, California, Texas, Idaho, Sonora, Michoacan, San Luis Potosi.. *torquata ochrorhyncha.*

EE. A yellow collar followed by a black one; scales 19–21, usually 21 (ventrals 162–174, caudals 43–53; Sinaloa, Durango, Colima, Tepic, Zacatecas, Nicaragua, Costa Rica).... *torquata torquata.*

BB. Banded snakes, the bands or rhombs reaching the first scale row or the ventrals; lateral spots absent or as darkening of the lateral spaces between the rhombs; body cylindrical.

CC. Scales 21–25; bands or rhombs.

D. Head markings present; dark mark from eye continuous or nearly so with first dorsal rhomb or band.

E. Rhombs; fangs grooved.

F. 18–23 rhombs reaching ventrals; no definite nape stripe (scales 21–23, usually 21; ventrals 181–187; caudals 66–81; end of Yucatan peninsula)..... *yucatanensis yucatanensis.*

FF. 25–39 rhombs reaching first scale row; nape stripe usually present (ventrals 174–191; caudals 64–81; scales 21; Campeche, Peten, Belize)..... *yucatanensis malleisi.*

EE. Bands.

F. 21–31 dark bands "at distances of from six to nine scales narrow cross-bands of one scale in width rise . . . . and meet or terminate in alternating positions, on or near the middle line of the back"; scales 23; ventrals 188; caudals 69; Vera Cruz..... *frenata.*

FF. 8–11 dark bands; scales 21–23; ventrals 186–189; caudals 66–80; fangs grooved or not; Guerrero and Puebla..... *latifasciata.*

DD. No head markings; rhombs on body.

E. Scales 21–23; ventrals 187–197; caudals 63–79; fangs grooved; Texas and Mexican Plateau to Tampico, San Luis Potosi and Zacualtipan..... *septentrionalis septentrionalis.*

EE. Scales 21–25; ventrals 160–186; caudals 55–74; fangs grooved; Vera Cruz, Zacualtipan, Sinaloa, Colima, Tepic, Guerrero, Oaxaca, Costa Rica..... *septentrionalis maculata.*

CC. Scales 19; bands.

D. 51–54 bands; fangs not grooved (ventrals 173–180; caudals 85–89; Oaxaca)..... *discolor.*

DD. 10–18 bands; fangs grooved.

E. 15–18 bands; ventrals 168–172; Nicaragua and Costa Rica..... *nigrofasciata.*

EE. 10–12 bands; ventrals 187–196; Guerrero and Oaxaca..... *mystacina.*

This key should be used with some care and attention, especially in distinguishing between *rhombifera* and *maculata*, *rhombifera* and *malleisi*, *rhombifera* and *polysticta*, *polysticta* and *septentrionalis*. No sound color characters separate *rhombifera* from *polysticta* or *maculata* from *septentrionalis*. Contrariwise *rhombifera* and *maculata* cannot well be separated on scalation, and neither can *polysticta* be told from *septentrionalis* on this basis.

Leptodeira Fitzinger

1843 *Leptodeira* Fitzinger, *Syst. Rept.*, p. 27 (type *annulata*).
1860 *Hypsiglena* Cope, *Proc. Acad. Nat. Sci. Philadelphia*, **12**, p. 246 (type *ochrorhyncha*).
1860 *Pseudodipsas* Peters, *Mon. Ak. Berlin*, p. 521 (type *torquata*).
1861 *Megalops* Hallowell, *Proc. Acad. Nat. Sci. Philadelphia*, **12**, p. 488 (type *maculata*).
1863 *Comastes* Jan, *Elenco Sist. Ofid.*, p. 102 (type *quincunciatus*).
1866 *Leptodira* Cope, *Proc. Acad. Nat. Sci. Philadelphia*, p. 127 (emendation).

Leptodeira annulata annulata (Linné)

1758 *Coluber annulatus* Linné, *Syst. Nat.* (10), **1**, p. 224.
1884 *Comastes ornata* Bocourt, *Bull. Soc. Philom.* (7), **8**, p. 141 (Darien).
1895 *Leptodira affinis* Günther, *Biol. Centr. Amer. Rept.*, p. 170 (Cent. Amer.).
1895 *Leptodira polysticta* Günther, loc. cit., p. 172 (in part, cotype BMNH 70–5–12, 6, Panamá).

Type: Existence questionable.
Type locality: America.
Range: South America to Nicaragua, giving place further north to the race *polysticta*.

Material: I have examined 101 (6 Nicaraguan, 17 Costa Rican and 78 Panamanian) specimens of this snake. Of the Panamanian specimens 57 were heads. I also have information on two Panamanian specimens in the Museum of the University of Michigan, and the published information on the two types of *ornata*. I have taken it on Barro Colorado Island, Panamá. It is a tree snake and is nocturnal. The northernmost record is from Cape Gracias, Nicaragua (USNM 15308–9). It reaches an elevation of 2000 meters at Tierra Blanca, Costa Rica (Coll. S. Luis Gonzaga). It occurs at sea level on both coasts.

NOTE: *Coluber albofuscus* Lacépède 1789 (*Hist. Nat. Serp.*, **2**, p. 94, 312) was considered by the describer to be the same as *annulata*, and stated to have the same ventral (190) and caudal (96) counts as Linné's species. Thus it is an absolute synonym of *annulata*, and was probably intended as a substitute name.

Leptodeira annulata polysticta Günther

1895 *Leptodira polysticta* Günther (in part), *Biol. Cent. Amer. Rept.*, p. 172, pl. 55, f. A.

Types: In Brit. Mus.

Type locality: Jalapa, Yucatan, Honduras, Belize, Panamá (Panamá type = *annulata annulata*).

Range: From Colima (one spec. in Mus. Mich.) and Tuxpan (USNM 25206–7, 25209–11) through Yucatan and Peten to eastern Honduras.

Material: I have seen 31 specimens (15 from Mexico, 1 from British Honduras, 5 from Guatemala, 10 from Honduras). The Colima specimen is the only one from the Pacific coast. I have information on the six cotypes in the BMNH and on 9 Peten specimens in the Michigan Museum.

Remarks: It is quite possible that specimens from Colima and Vera Cruz may represent a recognizable race, as they do not show the fine dotting which characterizes Yucatan specimens, and which is shown in Günther's figure. No local differences in scalation are evident. This race differs from *annulata* in that the dorsals are not enlarged, that a nape stripe is present, that the dorsal scales are usually 23 instead of 21 and that the ventrals range somewhat higher in count. The southernmost specimens come from the banana plantations around Tela and Lancetilla.

Leptodeira rhombifera Günther

1872 *Leptodira rhombifera* Günther, *Ann. Mag. Nat. Hist.* (4), **9**, p. 32.

1893 *Sibon septentrionale rubricatum* Cope, *Proc. Amer. Phil. Soc.*, **31**, p. 347 (Boca Mala, Costa Rica, type AMNH 17367).

1895 *Leptodira splendida* Günther, *Biol. Cent. Amer. Rept.*, p. 171, pl. 53, f. B (Izucar, Puebla, Mexico, types in Brit. Mus.).

1895 *Leptodira ocellata* Günther, loc. cit., p. 172, pl. 55, f. B (Chontales Mines, Nic., Cartago, Costa Rica. Types BMNH 94–10–1, 32 Chontales; BMNH 71–11–33, 11–13 Cartago).

Type: In Brit. Mus.

Type locality: Rio Chisoy, near Cubulco, Guatamala.

Range: From Plumosas, Sinaloa (USNM 46459) and Puebla (Mexico) into South America, apparently avoiding Yucatan and Peten. It occurs at Cartago, Costa Rica, at 4760 feet.

Material: 606 specimens. I have seen two Mexican specimens and have the information on the types of *splendida*. Twelve Guatemalan, two Salvadorian, fifteen

Honduranian, 28 Nicaraguan, 54 Costa Rican and 493 Panamanian specimens have been examined. Of the Panamanian specimens 342 were heads. Wettstein has reported on 5 from Bebedero, Costa Rica.

The type of *rubricata* (178 ventrals) is at one extreme of the range of variation, while a type of *ocellata* from Cartago (158 ventrals) is at the other. USNM 60027 from Empire, Canal Zone, also has 178, but aside from the above three the range of ventrals in counted specimens is 163–175. Of 78 specimens sixteen have 21, fifty-five have 23 and seven have 25 scale rows. Twenty-one scale rows seem more frequent in Mexico and in Panamá, whence no specimens with twenty-five rows are known.

Remarks: This is the form for which Boulenger resurrected the name *albofusca*, in my opinion incorrectly.

This species may eventually be divided into a northern form with large rhomboid spots (*rhombifera*) and a southern form with smaller squarish spots (*rubricata*). Honduranian specimens seem intermediate. I cannot, at present, divide the two accurately, as Mexican specimens seem identical with Costa Rican ones.

I have taken it at Las Canas, Costa Rica. It is a ground snake and nocturnal.

Leptodeira pacifica Cope

1866 ?? *Crotaphopeltis punctata* Peters, *Mon. Ak. Berlin*, p. 93 (?South Africa, type in Berlin Museum).

1869 *Leptodira pacifica* Cope, *Proc. Acad. Nat. Sci. Philadelphia*, **20**, p. 310.

Type: USNM 6833.

Type locality: Mazatlan, Mexico.

Range: Sinaloa and Jalisco, Mexico.

Material: USNM 6833, AMNH 19853 from Sinaloa, AMNH 19854 from Lerma river mouth, Lake Chapala, Jalisco and Boulenger's report on a specimen from Presidio, Sinaloa.

Remarks: I prefer to use Cope's name for this remarkably distinct little species rather than Peters', as I am not sure they are the same. Peters' locality adds to the uncertainty. His specimen had 19 scale rows, 151 ventrals, a divided anal and 65 caudals.

Leptodeira torquata torquata Günther

1860 *Leptodeira torquata* Günther, *Ann. Mag. Nat. Hist.* (3), **5**, p. 170, pl. 10, f. A.

1860 *Pseudodipsas fallax* Peters, *Mon. Ak. Berlin*, p. 521 (in synonomy of *L. torquata*).

1871 *Comastes quincunciatus* Jan, *Icon. Ofid.*, **38**, pl. 1, f. 1 (Mazatlan, Hamburg Mus.: Costa Rica, Vienna Mus.; Caracas, Heidelberg Mus.; Mexico, Vienna Mus., Westphalia Mus.; no locality, Milan Mus.).

1894 *Hypsiglena affinis* Boulenger, *Cat. Snakes Brit. Mus.* (2), p. 210, pl. 8, f. 1 (Zacatecas and Jalisco, types in Brit. Mus.).

Type: BMNH 61–12–30, 97.

Type locality: Laguna I., Nicaragua.

Range: From Costa Rica to Mazatlan, Sinaloa, and Ventanas, Durango.

Material: I have seen four specimens; the type in the British Museum, one from Colima (USNM 31285), one from San Blas, Tepic (USNM 51479) and one from Costa Rica (Museo Nacional).

Remarks: This is apparently a Pacific coast form. I consider Jan's Caracas locality erroneous. Boulenger's three types of *affinis* have a yellow collar and 19 scale rows, according to him. Günther says the Jalisco specimen has no yellow collar and the Zactecas specimens have 19 and 20 scale rows. I follow Günther in regarding the types of *affinis* as intermediates in color between *torquata* and *ochrorhyncha*.

Leptodeira torquata venusta (Moquard)

1899 *Hypsiglena venusta* Moquard, *Nouv. Arch. Mus. Paris* (4), **1**, p. 327.
Types: Six, in the Paris Museum.
Type locality: Santa Rosalia and San Ignacio, Baja California.
Range: Known only from type locality.
Material: None seen.

Remarks: The types were said to have from 178 to 186 ventrals. Moquard gives at the same time a range of 157–178 for *ochrorhyncha* from Baja California. Definite records and specimens I have seen indicate a range of 162 (a type of *affinis*) to 175 (type of *texana*) in the other forms of this species. Van Denburgh, however, without any details, says 160–191 for *ochrorhyncha*. As matters stand I prefer to maintain Moquard's species as a race of *torquata* although it seems to be surrounded by *ochrorhyncha*.

Leptodeira torquata ochrorhyncha (Cope)

1860 *Hypsiglena ochrorhynchus* Cope, *Proc. Acad. Nat. Sci. Philadelphia*, **12**, p. 246.
1860 *Hypsiglena chlorophaea* Cope, loc. cit., p. 247 (Cape San Lucas, Baja California. Types USNM 5283 and ANS 3537–8).
1893 *Hypsiglena texana* Stejneger, *N. Amer. Fauna*, **7**, 2, p. 205 (Laredo to Comargo, Texas. Type USNM 1782).
Type: USNM 4676, ANS 3748–9.
Type locality: Fort Buchanan, Arizona.
Range: Baja California, California, Nevada, Utah, Idaho, Arizona, New Mexico, Texas, Sonora, Chihuahua, Nuevo Leon, San Luis Potosi, Michoacan, Guanajuato.
Material: The seven types listed above; USNM 69538–40 from Cape San Lucas; USNM 46513 from Tupetaro, Michoacan; USNM 46444 from Rio Verde, San Luis Potosi; USNM 9889, 11369 from Guanajuato; USNM 14287 from Chihuahua. Fifteen in all.

Remarks: Günther's *torquata* was published in the April number of the *Ann. Mag.* which appeared, according to him, in February. Cope's *ochrorhyncha* was read in June, and had appeared by Nov. 15 according to the Index of the Academy publications. Thus *torquata* is earlier.

Leptodeira yucatanensis yucatanensis (Cope)

1866 *Leptodira annulata* var. Cope, *Proc. Acad. Nat. Sci. Philadelphia*, p. 127.
1887 *Sibon annulata yucatanensis* Cope, *Bull. U. S. N. M.* 32, p. 67.
Type: USNM 24887.
Type locality: Yucatan.

Range: Yucatan (only definite locality Chicheń Itza).
Material: The type and eight specimens from Chicheń Itza.

Remarks: Cope described, but did not name, this form in 1866. In 1887 he named, but did not describe it, merely referring to his 1866 paper. This form is a remarkably colored Leptodeira, but it apparently intergrades with *malleisi*, which is close to and may intergrade with *rhombifera*. Should this be proved, both *yucatanensis* and *malleisi* will become races of *rhombifera*.

Leptodeira yucatanensis malleisi Dunn and Stuart

1935 *Leptodeira yucatanensis malleisi* Dunn and Stuart, *Occ. Papers Mus. Zoöl. U. Michigan*, **313**, p. 1.
Type: U. Mich. 73230.
Type locality: Tuxpena, Campeche.
Range: Campeche, Peten, British Honduras.
Material: The type and fifteen paratypes mentioned in the original description.

Remarks: Differs from *yucatanensis* only in color, and by this difference it more nearly resembles *rhombifera*.

Leptodeira frenata (Cope)

1886 *Sibon frenatum* Cope, *Proc. U. S. Nat. Mus.*, **9**, p. 184.
Type: Not known to exist.
Type locality: Jalapa, Mexico.
Range: Known only from type locality.
Material: None seen.

Remarks: Cope based this species on a very small specimen, 305 mm. in total length. It cannot now be found. No other banded Leptodeira is known in which the light bands are only one scale in width, and only one other banded species is known with the dark postocular mark reaching the first dorsal band, and this, *latifasciata*, has only about half as many bands.

Leptodeira latifasciata (Günther)

1894 *Hypsiglena latifasciata* Günther, *Biol. Cent. Amer. Rept.*, p. 138, pl. 49, f. B.
1905 *Leptodira guilleni* Boulenger, *Proc. Zool. Soc. London*, **2**, p. 247, pl. 7, f. 2 (type in Brit. Mus., Rio Balsas, Guerrero).
Type: In British Museum.
Type locality: Southern Mexico.
Range: Guerrero and Puebla.
Material: USNM 46550, from Piaxtla, Puebla.

Remarks: The specimen from Puebla has no visible grooves on the fangs. The three specimens listed below seem to me conspecific despite the fact that two are apparently without grooved fangs while one has grooves.

	SCALES	VENTRALS	CAUDALS	BANDS	GROOVED FANGS
Type *latifasciata*	21	186	80	10	No
Type *guilleni*	23	189	71	11	Yes
USNM 46550	23	181	66	8	No

Leptodeira septentrionalis septentrionalis (Kennicott)

1859 *Dipsas septentrionalis* Kennicott in Baird, *Report Mex. Bound. Surv.*, **2**, Rept., p. 16, pl. 8, f. 1.

Types: USNM 4267 (two, Matamoros), 2288 (Brownsville, not extant).

Type locality: Matamoros, Mex., and Brownsville, Texas.

Range: Cameron Co., Texas, to Tampico (U. Mich. 3791), San Luis Potosi (M.C.Z. 4516) and Zacualtipan (A.N.S. 11662–3, 14775).

Material: Fourteen specimens have been seen.

Remarks: Indistinguishable in color from the more southern *maculata*, but has more ventrals. Both seem to occur at Zacualtipan, but specimens labeled as from that locality may have come from very different altitudes.

Leptodeira septentrionalis maculata (Hallowell)

1861 *Megalops maculatus* Hallowell, *Proc. Acad. Nat. Sci. Philadelphia*, 1860, p. 488.

1869 *Leptodira personata* Cope, *Proc. Acad. Nat. Sci. Philadelphia*, **20**, p. 310 (Type USNM 6836, Mazatlan, Mex.).

1913 *Leptodira dunckeri* Werner, *Mitt. Nat. Mus. Hamburg*, **30**, p. 28 (type in Hamburg; Mexico or Venezuela).

Type: USNM 7367, collected by Rogers Exped.

Type locality: Tahiti [in error].

Range: Cape San Lucas (USNM 11290), Mazatlan (type of *personata*), Zacualtipan (ANS 14774), Tuxpan (USNM 25208) to Oaxaca (Tepanatepec, MCZ 27732–98, and Zamtepec, USNM 46497–8), Nicaragua (?, type of *maculata*), Costa Rica (six specimens in Museo Nacional).

Material: I have examined 133 specimens. In addition the descriptions of *dunckeri*, of eleven Jalapa specimens in the British Museum and of one from Vera Cruz in the University of Michigan have been used.

Remarks: The type of *maculata* agrees almost exactly with the type of *personata*. As all the Rogers material from Central America was collected in Nicaragua, it probably came from that country. Aside from six in the Museo Nacional de Costa Rica, no others are known south of Tehuantepec. As nothing similar to *dunckeri* is known from Venezuela, and as the description shows no differences from *maculata*, I regard the Mexican alternative locality as almost certain. This form seems to replace *septentrionalis* off the plateau in Mexico. I have taken it at Jalapa. It is terrestrial.

Leptodeira discolor Günther

1860 *Leptodeira discolor* Günther, *Proc. Zool. Soc. London*, p. 317.

Types: In Brit. Mus. (two specimens).

Type locality: Oaxaca, Mexico.

Range: Known only from type locality.

Material: None seen.

Remarks: The two cotypes are the only specimens known of this apparently very distinct form. Its nearest ally would seem to be some form like *maculata*, rather than either of the other two species with ungrooved fangs.

Leptodeira mystacina Cope

1870 *Leptodira mystacina* Cope, *Proc. Amer. Phil. Soc.*, **11**, p. 151.
Types: USNM 30339–40.
Type locality: Near Isthmus of Tehuantepec, Mexico.
Range: Known definitely only from Acapulco, Guerrero (USNM 46551), and Tapanatepec, Oaxaca (MCZ 27835–6).
Material: The types and three additional specimens.

Leptodeira nigrofasciata Günther

1868 *Leptodeira nigrofasciata* Günther, *Ann. Mag. Nat. Hist.* (4), **1**, p. 425.
Type: BMNH 66–12–65.
Type locality: Nicaragua.
Range: Nicaragua (Managua, USNM 79967–70); Costa Rica (Turrialba, Collegio San Luis Gonzaga); west coast of Central America (ANS 11313–15).
Material: 19 specimens, 13 from Nicaragua and two from Costa Rica.

* Contribution No. 26.

ON SOME NEW INVARIANTS OF A MANIFOLD

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In the present Note we consider the minimum number μ_i of elements of a certain type into which a given n-manifold M_n may be decomposed. The manifolds considered are manifolds in the sense of Newman-Alexander.* The number μ_i depends, of course, upon the type of element considered. The various characters thus obtained and their comparison lead one to consider interesting properties, related to other questions on the topology of manifolds.

We have considered three different types of elements. The simplest is the equivalent of a simplex from the standpoint of combinatorial analysis situs; namely, a manifold whose matrices of incidence are identical with those of a subdivision of a simplex into polyhedra. The second type of element is a manifold with regular boundary, whose homology groups are the same of those of a simplex. The third type of element has the same definition as the second, but with the difference that the homology groups are referred to the whole fundamental manifold, and not merely to the

element itself. The subdivision of the manifold into elements of the first or second type leads, of course, to an ordinary or combinatorial complex.

Calling μ_0, μ_1, μ_2 the minimum number of elements of the subdivisions of a given manifold, respectively, into elements of the first, second and third type, I prove the following theorem:

In an absolute n*-manifold* $2 \leqq \mu_i \leqq n + 1$ $(i = 0, 1, 2)$, *and* μ_i *can have all the integer values between these limits. In a manifold with regular boundary the same results are valid if* 2 *and* $n + 1$ *are replaced respectively by* 1 *and* n.

In the case of $i = 1, 2$ the same results are valid also for combinatorial manifolds.

If $i = 0$, one deduces the following result of combinatorial topology:

Every n*-manifold without boundary can be obtained by means of identifications of the faces of* $n + 1$ *convex polyhedra* (two identified faces being on different polyhedra). *If one operates in the same manner on a number of polyhedra less than* $n + 1$ *one cannot obtain the most general manifold. In the analogous manner, by means of* n n*-polyhedra and not less, one can construct every* n*-manifold with regular boundary.*

Among other results we obtain the relation $\mu_i(A \times B) \leqq \mu_i(A) + \mu_i(B) - 1$ ($A \times B$ being the direct product of the manifolds A and B), where in some cases the equality holds.

The differences $r_1 = \mu_0 - \mu_1$ and $r_2 = \mu_0 - \mu_2$ are particularly interesting. These characters are zero in the most common manifolds and are $\neq 0$ in the spaces of Poincaré (manifolds which are combinatorial spheres or cells) which are not ordinary spheres or cells. Therefore it seems that the vanishing of these characters indicates the presence in a general manifold of properties analogous to those which differentiate the most general spaces of Poincaré from the ordinary cells or spheres.

A complete description of the properties of μ_0 is at present very difficult, because it involves some unsolved problems of the topology of manifolds. The least difficult character to study is μ_2. I observe that, if the homologies considered in the definition of μ_2 are mod.2, μ_2 coincides with a known invariant, considered in the Calculus of Variations, the *combinatorial category of Schnirelmann.* There are relations between μ_2 and the tensors of Alexander, attached to the multiple intersections of cycles of the manifold, useful in studying μ_2. In the case of the combinatorial category of an absolute manifold this invariant is the maximum rank of these tensors relative to cycles mod.2; namely, the maximum number of cycles mod.2, whose combinatorial intersection is not null.

* M. H. A. Newman, *Amsterdam Proc.*, **29**, 611-626, 627–641 (1926); **30**, 670–673 (1927). J. W. Alexander, *Ann. Math.* (2), **31**, 294–322 (1930).

ENUMERATION OF THE ABELIAN GROUPS WHOSE ORDERS DO NOT EXCEED A GIVEN NUMBER

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Methods for the enumeration of all the abelian groups of a given order g are well known but nothing seems to have been done to develop practical methods for the enumeration of all the abelian groups whose orders do not exceed a given number n. It is known that there is one and only one cyclic group of every order g and hence there are n cyclic groups whose orders do not exceed n if we include the identity among these groups, as will be done in what follows. We shall therefore confine our attention hereafter to the non-cyclic abelian groups unless the contrary is explicitly stated. The object is to determine formulas for the enumeration of these groups so that the number of them can be found directly by substituting in these formulas the appropriate numerical values and hence the use of these formulas will not involve any group theoretic considerations. The method for determining these formulas may be briefly described as follows:

First determine the number of the non-cyclic abelian groups which satisfy certain given conditions and whose orders do not exceed n. Then determine the number of additional groups which satisfy these conditions and some other condition. Continue this process until all the conditions which affect the number of the possible non-cyclic abelian groups have been exhausted. For instance, there is one non-cyclic abelian group of order p^2, p being a prime number. Hence there are at least $[n/p^2]$ non-cyclic abelian groups whose orders do not exceed n, where the symbol $[n/p^2]$ implies the number of natural numbers which do not exceed n/p^2, as usual. As there are two non-cyclic abelian groups of order p^3 the number of the non-cyclic groups whose orders do not exceed n is at least the sum of the two numbers $[n/p^2]$ and $[n/p^3]$. In a similar manner we see that this number is at least as large as the following expression:

$$\left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + 2\left[\frac{n}{p^4}\right] + 2\left[\frac{n}{p^5}\right] + 4\left[\frac{n}{p^6}\right] + 4\left[\frac{n}{p^7}\right] + 7\left[\frac{n}{p^8}\right] + 8\left[\frac{n}{p^9}\right] + \ldots$$

until we arrive at the highest power of the prime p which does not exceed n, and where p represents successively all the different prime numbers whose squares do not exceed n.

The coefficients 1, 1, 2, 2, ... result directly from the number of the partitions with respect to addition of the corresponding exponent of p and hence they can be readily determined for small values of these exponents. It remains to determine the additional non-cyclic abelian groups when n is at least equal to the product of the squares of more than one prime number. If n is not as large as the square of a given number this number requires no consideration, since it does not affect the number of the non-cyclic abelian groups whose orders do not exceed n. Since there are three non-cyclic abelian groups of order $p_1^2p_2^2$, p_1 and p_2 being prime numbers, and one of these groups for each of these two primes was considered in the formula noted in the preceding paragraph, it results that the expression $[n/p_1^2p_2^2]$ has unity for its coefficient. Similarly, it results that the expression $[n/p_1^2p_2^2 \ldots p_l^2]$ has unity for its coefficient. Also whenever l exceeds 2, since the number of the non-cyclic abelian groups of order $p_1^2 p_2^2 \ldots p_l^2$ is $2^l - 1$ and the number of the combinations of $p_1^2, p_2^2, \ldots, p_l^2$, taken one at a time, two at a time, up to $l - 1$ at a time is $2^l - 2$.

There are five non-cyclic abelian groups of order $p_1^3p_2^2$. As four of these were considered under the preceding formulas the expression $[n/p_1^3p_2^2]$ has unity for its coefficient. The expression $[n/p_1^3p_2^2 \ldots p_l^2]$ has the same coefficient since the number of the non-cyclic abelian groups of order $p_1^3p_2^2 \ldots p_l^2$ is equal to the number of the non-cyclic abelian groups of order $p_1^2 p_2^2 \ldots p_l^2$ increased by 2^{l-1} but the number of combinations of 2, 3, . . , l taken one, two, $l - 1$ at a time is $2^{l-1} - 2$ and one must be added to this because $[n/p_1^3]$ has the coefficient unity in the first formula. From similar considerations it results that in order to obtain the total number of the non-cyclic abelian groups whose orders do not exceed n it is necessary to add to the formula given in the second paragraph expressions like the following:

$$\left[\frac{n}{p_1^2p_2^2 \ldots p_l^2}\right] + \left[\frac{n}{p_1^3p_2^2 \ldots p_l^2}\right] + \left[\frac{n}{p_1^3p_2^3}\right] + 2\left[\frac{n}{p_1^4p_2^2}\right] + 2\left[\frac{n}{p_1^4p_2^3}\right] +$$

$$2\left[\frac{n}{p_1^5p_2^2}\right] + 2\left[\frac{n}{p_1^5p_2^3}\right] + 4\left[\frac{n}{p_1^6p_2^2}\right] + \ldots$$

It is easy to extend these formulas by the determination of the appropriate coefficients. These extensions may be made by starting with various terms which precede, and the work involved can be lessened by a suitable choice of these starting terms. The terms actually given above suffice to determine the number of the non-cyclic abelian groups whose orders do not exceed n when n is any number up to 1000 at least. The numbers of these groups when $n = 200$ is readily found to be 189 which is less than the number of the cyclic groups whose orders do not exceed 200. The numbers of these groups whose orders do not exceed 1000 is found, by substituting in

the given expressions, to be 1091, which is a little more than the number of the cyclic groups whose orders do not exceed 1000. The total number of the abelian groups whose orders do not exceed 200 is therefore 389 and the total number of these groups whose orders do not exceed 1000 is 2091.

The total number of the abelian groups whose orders do not exceed a given number n cannot exceed the sum of these orders and it is equal to this sum only in the trivial case when $n = 1$. When n exceeds 5 it cannot be as large as half this sum, but no close approximations to this number when n is large seem to be known. If every subgroup of an abelian group is cyclic the group itself is cyclic and the quaternion group is the only group which has the property that it is non-cyclic but contains only cyclic subgroups. The total number of the abstract groups of a given finite order g can obviously not exceed $(g!)^{g-1}$ since each such group can be represented as a regular substitution group and hence it is a subgroup of the symmetric group of degree g. The number of these groups can therefore not exceed the number of the different combinations, g at a time, of the substitutions of the symmetric group. While this constitutes a simple proof of the fact that the number of the distinct groups of a given order is finite it obviously does not furnish a close approximation to the number of such groups when g is large. No close approximation seems to be known.

In Crelle's *Journal*, volume 175 (1936), page 252, a prize is offered for the determination of all the abstract groups whose orders do not exceed 200. It was noted above that there are 389 such abelian groups. Among the non-abelian ones those which contain a cyclic subgroup of index 2 are very elementary but somewhat numerous. There are obviously $2^k - 1$ such groups when this cyclic subgroup is of odd order, where k is the number of the distinct prime numbers which divide this order. When the order of this cyclic subgroup is twice an odd number the number of these non-abelian groups is $2^k - 2$ and when it is four times an odd number there are $2^{k+1} - 2$ such groups. Finally, when the order of this cyclic group is divisible by 8 the number of these groups is $3.2^k - 2$. Hence there are 347 non-abelian groups whose orders do not exceed 200 but which separately involve a cyclic subgroup of index 2. The total number of the groups whose orders do not exceed 200 and which are either abelian or are non-abelian but contain a cyclic subgroup of index 2 is therefore 736.

The number of the non-abelian groups which involve invariantly a given cyclic subgroup of index p, p being an odd prime number, is unity whenever the order of this cyclic subgroup is prime to p and is divisible by one and only one prime number which is congruent to unity modulo p or is not divisible by such a prime number but is divisible by p^2, because the order of the group of isomorphisms of a cyclic group whose order is a power of 2 is not divisible by an odd prime number and the group of isomorphisms of a cyclic group whose order is a power of an odd prime number is cyclic

When the order of the given invariant cyclic subgroup is prime to p and is divisible by $k > 1$ distinct prime numbers which are separately congruent to unity modulo p then the number of these groups is $p^{k-1} + p^{k-2}$. When the order of this cyclic subgroup is divisible by p but not by p^2 this number is $2(p^{k-1} + p^{k-2})$ and when it is divisible by p^2 it is $2(p^{k-1} + p^{k-2} + p^k + p^{k-1}$. Finally, when this order is divisible by one and only one prime number which is congruent to unity modulo p and is also divisible by p but not by p^2 there are two such groups, and when it is also divisible by p^2 the number of these non-abelian groups is $p + 2$.

Hence the total number of the non-abelian groups which contain invariantly a cyclic subgroup of odd prime index p is unity in each of the two cases when the order of this cyclic subgroup is either prime to p and involves one and only one prime factor which is congruent to unity modulo p or when it involves no such prime factor but is divisible by p^2. There are two and only two such groups when the order of this cyclic subgroup is divisible by p but not by p^2 and involves one and only one prime factor which is congruent to unity modulo p. There are $p + 2$ such groups when this order is divisible by p^2 and involves one and only one prime factor which is congruent to unity modulo p. When it involves $k > 1$ such prime factors the number of these groups is $p^{k-1} + p^{k-2}$, $2(p^{k-1} + p^{k-2})$, or $p^k + 3p^{k-1} + 2p^{k-2}$ as the order of the given cyclic subgroup is prime to p, is divisible by p but not by p^2, or is divisible by p^2. Hence the number of these groups whose orders do not exceed 200 is 40 and we have found here 776 of the abstract groups whose orders do not exceed 200.

GROUPS WHOSE PRIME POWERS GENERATE A CYCLIC SUBGROUP

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The subgroup generated by the squares of the operators of a group G includes the commutator subgroup of G but the subgroup generated by the pth powers of the operators of G, p being an odd prime number, does not necessarily include the commutator subgroup of G. Hence there is a great difference between the study of the groups when the subgroup generated by the squares of its operators is given and when the subgroup generated by its pth powers, p being an odd prime number, is given. If in the latter case we add the condition that this subgroup includes the commutator

subgroup much of this difference will disappear and hence we shall assume in what follows that the cyclic subgroup generated by the prime powers includes the commutator subgroup of G. Since the case when this prime is even has received considerable attention it will be assumed in what follows, unless the contrary is stated, that the prime is odd. It is obvious that the order of such a group is of the form p^m.

When G is abelian it is of type $\alpha + 1, 1^n$, p^α being the order of the cyclic subgroup generated by the pth powers of the operators of G. Since this requires no further consideration here we shall assume in what follows that G is non-abelian. Each of its cyclic subgroups of order $p^{\alpha+1}$ is invariant since the given cyclic subgroup of order p^α contains the commutator subgroup of G. The order of the commutator subgroup of G is p because G is generated by its operators of order $p^{\alpha+1}$. It is therefore conformal with the abelian group of type $\alpha + 1, 1^n$ and contains non-invariant operators of order $p^{\alpha+1}$. It contains also non-invariant operators of order p^β, $0 < \beta < \alpha + 1$ since two of its non-commutative operators of order $p^{\alpha+1}$ generate a group of order $p^{\alpha+2}$ which involves non-invariant operators of order p^β. That is, *if the operators of a non-abelian group of order* p^m *generate a cyclic group of order* p^α *which includes the commutator subgroup then the group contains non-invariant operators of each of the orders* $\mathrm{p}, \mathrm{p}^2, \ldots, \mathrm{p}^{\alpha+1}$.

There are two cases to be considered. In one of these G contains invariant operators of order $p^{\alpha+1}$ while in the other it does not have this property. In the latter case it contains a subgroup of index p which has this property and hence we shall first consider the former case. All the operators of G which are commutative with a non-invariant operator s_1 of order p constitute a subgroup of index p which is conformal with the abelian group of type $\alpha + 1, 1^n$. The operators of G which do not appear in this subgroup include a non-invariant operator s_2 of order p which is also invariant under a subgroup of index p. The cross-cut of these two subgroups of index p is an invariant subgroup of index p^2 under G which is composed of operators which are commutative with each of the two non-commutative operators s_1, s_2 and includes invariant operators of order $p^{\alpha+1}$. If this cross-cut is non-abelian we proceed as before until we arrive at such an abelian cross-cut which is of type $\alpha + 1, 1^n$ and whose central is the original central of G extended by the group generated by $s_1, s_2, \ldots$

From the preceding paragraph it results that all the non-abelian groups of order p^m, p being an odd prime number, which have the property that the pth powers of their operators generate a cyclic group of order p^α which includes the commutator subgroup and involves invariant operators of order $p^{\alpha+1}$, can be constructed as follows: Start with an abelian group of type $\alpha + 1, 1^n$, $n \geqq (m - \alpha - 1)/2$, and adjoin to it an operator of order p which transforms an operator of order p of this abelian group into itself

multiplied by an operator of order p generated by an operator of order $p^{\alpha+1}$ contained in the given abelian subgroup. This adjoined operator is assumed to be commutative with a subgroup of index p of the given abelian subgroup and hence this subgroup of index p involves operators of order $p^{\alpha+1}$.

When $m > n + \alpha + 2$ an operator of order p is adjoined to the group thus obtained such that this operator of order p transforms an operator of order p in its central into itself multiplied by an operator of order p in its commutator subgroup. The second adjoined operator of order p may be assumed to be commutative with each of the two non-commutative operators of order p noted in the preceding paragraph and the central of the resulting group is of index p^2 under the abelian group with which we started and it contains operators of order $p^{\alpha+1}$. When $m > n + \alpha + 3$ the given operations can be repeated until a group of order p^m is reached. Hence the following theorem has been established: *There is one and only one group of order* p^m *whose central is a given abelian group of type* $\alpha + 1, 1^{n-k}$, *whenever* m *exceeds* $n + \alpha$ *by an even natural number,* $2k$, *such that the* p*th powers of the operators of this group generate a cyclic group of order* p^α *which includes the commutator subgroup of the group.*

When α is fixed n may be any given natural number and the number of the distinct groups obtained by the given method is n. All of these groups have centrals of different orders whose types vary from $\alpha + 1, 1^{n-1}$ to $\alpha + 1$. Whenever the prime powers of the operators of a given group generate the cyclic subgroup of order p^α then the prime powers of the direct product of this group and the abelian group of type 1^n has the same property. To simplify the considerations which follow such direct products will henceforth be excluded and hence we shall consider only groups whose centrals are either the cyclic group of order $p^{\alpha+1}$ or the cyclic group of order p^α. The other possible groups are the direct products of such groups and the abelian group of type 1^n. All such groups of order p^m whose centrals include operators of order $p^{\alpha+1}$ can therefore be constructed by starting with the cyclic group of order $p^{\alpha+1}$ and forming the direct product of this group and a group of order p. This direct product is then extended by an operator of order p which transforms a generator of the second of the given generating groups into itself multiplied by an operator of order p contained in the first.

When m exceeds $\alpha + 3$ this process is repeated and hence there is one and only one such group of order $p^{\alpha+1+2k}$ where k is an arbitrary natural number. When the central of G is p^α there is a subgroup of index p in G whose central is $p^{\alpha+1}$. Hence such a G can be obtained by extending this subgroup by an operator of order p which is commutative with all the operators whose orders are less than $p^{\alpha+1}$ in its central. In order to see that

this condition determines the isomorphism it is desirable to stress the fact that if a group of order p^m has a commutator subgroup of order p then every one of its subgroups of index p which includes its central is composed of operators which are invariant under one of its inner isomorphisms. This theorem results directly from the facts that the number of such subgroups is equal to the number of these isomorphisms and that no two distinct isomorphisms could correspond to the same such subgroup.

It therefore results that every such group of order p^{m-1} which has the property that its central is the cyclic group of order $p^{\alpha+1}$ can be extended by an operator of order p so as to obtain a group of order p^m such that the pth powers of its operators generate the same cyclic group of order p^α including the commutator subgroup but containing a central of order p^α. As this extension can be effected in essentially only one way there results the following theorem: *There is one and only one group of order* p^{m}, $\mathrm{m} > \alpha$ *and* p *being an odd prime number, which is not a direct product of two groups and satisfies the conditions that the* p*th powers of its operators generate the cyclic group of order* p^{α} *which includes the commutator subgroup of the group.* When $m - \alpha$ is odd the central of this group is the cyclic group of order $p^{\alpha+1}$ and when $m - \alpha$ is even it is the cyclic group of order p^α.

From what precedes it results that the consideration of the groups which have the property that the pth powers of their operators generate a cyclic group including their commutator subgroup is much simpler when p is an odd prime number than when it is equal to 2. This is largely due to the fact that when p is an odd prime number then the order of the product of two non-commutative operators of a group of order p^m whose commutator subgroup is of order p is the same as if these two operators were commutative while this is not always the case when $p = 2$. If the condition that the cyclic group generated by the pth of the operators of the group includes the commutator subgroup is omitted then the case when $p = 2$ is much simpler than the case when p is odd.

[1] G. A. Miller, *Proc. Nat. Acad. Sci.*, **22**, 288–291 (1936).

ON REGULAR RINGS

BY JOHN VON NEUMANN

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Introduction. 1. In what follows we shall consider rings $\mathfrak{R}$ (which are associative but not necessarily commutative) with unit 1—cf. v.d.W.[1] I., pp. 37–40. The theory of *semi-simplicity* of these $\mathfrak{R}$ has always been carried out on the basis "Chain-condition" or "Minimum-condition"—cf. v.d.W. II, p. 151, for semi-simplicity *Ibid.*, pp. 156–172. The object of this note is to give a theory of "regularity" of $\mathfrak{R}$, this notion being equivalent to semi-simplicity when the chain-condition is fulfilled, but possessing most essential features of the semi-simple theory quite independently of the chain-condition. In a subsequent note we will use these results to establish connections between abstract algebra and the "continuous geometries" introduced by the author in two earlier notes in these PROCEEDINGS.

It is defined by a simple algebraical condition, which seems to be new even as a criterion of semi-simplicity (when the chain-condition is required). Our method is purely algebraical, but a greater stress is laid on the lattice-theoretical[2] aspect of right- and left-ideals.

Definition of Regularity. 2. We define as usual (v.d.W. II, pp. 53–54):

Definition 1. A right (left) ideal—abbreviated: r.(l.)i.—is a set $\mathfrak{a} \subset \mathfrak{R}$ such that $\alpha)$ $x,y \in \mathfrak{a}$ imply $x + y \in \mathfrak{a}$, $\beta)$ $x \in \mathfrak{a}$, $z \in \mathfrak{R}$ implies $xz \in \mathfrak{a}$ $(zx \in \mathfrak{a})$. If $a \in \mathfrak{R}$ then a unique minimal r.(l.)i. exists, which contains a: The *principal right (left) ideal*—abbreviated: p.r.(l.)i.—of a, that is the set $(a)_r$ $((a)_l)$ of all az (za), $z \in \mathfrak{R}$.

All statements we make for r. and for l.i. remain true if we interchange r. and l.

The following statements are evident: The r.i. form a partially ordered set with respect to set-theoretical inclusion $\mathfrak{a} \subset \mathfrak{b}$. This set has a minimum element: $(0) = (0)_r$ and a maximum one: $\mathfrak{R} = (1)_r$. For any set of r.i. $\mathfrak{a},\mathfrak{b},\ldots$ a maximum r.i. $\subset \mathfrak{a},\mathfrak{b},\ldots$ exists: The set-theoretical intersection of $\mathfrak{a},\mathfrak{b},\ldots$—*gr.l.b.*$(\mathfrak{a},\mathfrak{b},\ldots)$. Similarly a minimum r.i. $\supset \mathfrak{a},\mathfrak{b},\ldots$ exists: The set-theoretical intersection of all r.i. $\supset \mathfrak{a},\mathfrak{b},\ldots$—*l.u.b.*$(\mathfrak{a},\mathfrak{b},\ldots)$.

Definition 2. Write $\mathfrak{a} \cup \mathfrak{b} = gr.l.b.\ (\mathfrak{a},\mathfrak{b})$, $\mathfrak{a} \cup \mathfrak{b} = l.u.b.\ (\mathfrak{a},\mathfrak{b})$. Thus the r.i. form a *lattice* with *meet* $\mathfrak{a} \cap \mathfrak{b}$ and *join* $\mathfrak{a} \cup \mathfrak{b}$, *zero* (0), *unit* $\mathfrak{R}$ (cf. G. Birkhoff[2] p. 442) Two r.i. $\mathfrak{a},\mathfrak{b}$ are *inverses* if $\mathfrak{a} \cap \mathfrak{b} = (0)$, $\mathfrak{a} \cup \mathfrak{b} = \mathfrak{R}$. (Similarly for l.i.)

Clearly $\mathfrak{a} \cup \mathfrak{b}$ is the set of all $x + y$, $x \in \mathfrak{a}$, $y \in \mathfrak{b}$.

Definition 3. An element $e \in \mathfrak{R}$ is *idempotent*—abbreviated: ip.—if $e^2 = e$.

We establish some basic properties of these notions.

Lemma 1. e is ip. if, and only if, $1 - e$ is ip.—*Proof:* $e^2 = e$ means $e(1 - e) = 0$ which is symmetric in e and $1 - e$.

Lemma 2. For an ip. e, $x \epsilon (e)_r$ means $ex = x$.—*Proof:* As $ex \epsilon (e)_r$, the condition is sufficient; as $x \epsilon (e)_r$ implies $x = ey$, $ex = e^2y = ey = x$, it is necessary.

Lemma 3. $\mathfrak{a}, \mathfrak{b}$ are inverse r.i. if and only if $\mathfrak{a} = (e)_r$, $\mathfrak{b} = (1 - e)_r$ for a suitably chosen ip. e.—*Proof:* Sufficiency: As $1 = e + (1 - e) \epsilon \mathfrak{a} \cup \mathfrak{b}$ so $\mathfrak{a} \cup \mathfrak{b} = \mathfrak{R}$. $x \epsilon \mathfrak{a} \cap \mathfrak{b}$ implies $ex = x$ and $(1 - e)x = x$, $ex = 0$, so $x = 0$. Hence $\mathfrak{a} \cap \mathfrak{b} = (0)$.—Necessity: Let $\mathfrak{a}, \mathfrak{b}$ be inverse r.i. Then $1 = x + y$, $x \epsilon \mathfrak{a}$, $y \epsilon \mathfrak{b}$. If $z \epsilon \mathfrak{a}$ then $z = xz + yz$, and $xz \epsilon \mathfrak{a}$ as $x \epsilon \mathfrak{a}$, so $yz \epsilon \mathfrak{a}$. But $yz \epsilon \mathfrak{b}$ as $y \epsilon \mathfrak{b}$ so $yz \epsilon \mathfrak{a} \cap \mathfrak{b}$, $yz = 0$. So $z = xz \epsilon (x)_r$. Thus $\mathfrak{a} \subset (x)_r$. Clearly $(x)_r \subset \mathfrak{a}$, so $\mathfrak{a} = (x)_r$. Similarly $\mathfrak{b} = (y)_r = (1 - x)_r$. Finally $x - x^2 = x(1 - x) = (1 - x)x \epsilon \mathfrak{a}$ and $\epsilon \mathfrak{b}$, so $\epsilon \mathfrak{a} \cap \mathfrak{b}$ so $= 0$. Thus x is ip., and is the desired e.

Lemma 4. The above $\mathfrak{a}, \mathfrak{b}$ determine the ip. e uniquely.—*Proof:* That is: $(e)_r = (f)_r$, $(1 - e)_r = (1 - f)_r$, e, f ip., imply $e = f$. The first relation gives $e \epsilon (f)_r$, $fe = e$, $(1 - f)e = 0$. Replace e, f by $1 - e$, $1 - f$, so the second relation gives $f(1 - e) = 0$, $fe = f$. Hence $e = f$.

Lemma 5. $(a)_r = (e)_r$, e an ip., is equivalent to this: An x with $axa = a$ exists, and $e = ax$.—*Proof:* The condition is: $e \epsilon (a)_r$, e an ip., $a \epsilon (e)_r$. The first requirement means: $e = ax$; the second one: $e^2 = e$, that is $axax = ax$. Now the third one becomes: $ea = a$, that is $axa = a$. Since this implies the preceding equation, we have only $axa = a$, $e = ax$ left.

Lemma 6. The three following properties of a are equivalent to each other: α) An x with $axa = a$ exists. β) An ip. e with $(a)_r = (e)_r$ exists. γ) An r.i. $\mathfrak{b}$ which is inverse to $(a)_r$ exists.—*Proof:* β), γ) are equivalent by Lemma 3, α),β) are equivalent by Lemma 5.—

We now define:

Definition 4. $\mathfrak{R}$ is *regular* if for every $a \epsilon \mathfrak{R}$ an x with $axa = a$ exist. By Lemma 6, and owing to the symmetry of the above statement with respect to r. and l., it is equivalent to each one of the following conditions: α) An ip. e with $(a)_r = (e)_r$ exists. β) An ip. f with $(a)_l = (f)_l$ exists. γ) An r.i. $\mathfrak{b}$ which is inverse to $(a)_r$ exists. δ) An l.i. $\mathfrak{c}$ which is inverse to $(a)_l$ exists.

The form α) of our regularity-definition makes it clear that it coincides with semi-simplicity whenever the chain-condition holds. Indeed: Assume α). If $\mathfrak{a}$ is an r.i. $\neq (0)$ choose an $a \neq 0$ with $a \epsilon \mathfrak{a}$ and an ip. e with $(a)_r = (e)_r$. Then $a \neq 0$ gives $e \neq 0$ and $e \epsilon (e)_r = (a)_r = \mathfrak{a}$, $e = e^n \epsilon \mathfrak{a}^n$, so $\mathfrak{a}^n \neq (0)$, $\mathfrak{a}$ is not nilpotent. Conversely, assume the semi-simplicity of $\mathfrak{R}$ and the chain-condition. Then every "minimum" r.i. of $\mathfrak{R}$ fulfils α) by v.d.W. II, p. 157 ("Hilfssatz 3"), and this extends to all r.i. by the method *Ibid.*, p. 158 ("Satz 1," replace $\mathfrak{o} = \mathfrak{R}$ by the r.i. in

question). We see, incidentally, that all r.i. in $\mathfrak{R}$ are principal, which is not generally true without the chain-condition.

It seems to be worth emphasizing that our definition of regularity by $axa = a$ is obviously r.-l.-symmetric, which the usual definition of semi-simplicity is not, and that it exhibits a remarkable similarity to the requirement of the existence of an "inverse element" ($ax = xa = 1$) in division-algebras.

Principal Right- and Left-Ideals. 3. From now on we assume $\mathfrak{R}$ to be regular, and denote the sets of all p.r.(l.)i. by $R_{\mathfrak{R}}$ ($L_{\mathfrak{R}}$). We will establish an important correspondence between $R_{\mathfrak{R}}$ and $L_{\mathfrak{R}}$. Again all statements arising from the ones which we will formulate by interchanging r. and l. will be true too.

Definition 5. If $\mathfrak{a}$ is an r.(l.)i., then let $\mathfrak{a}^l(\mathfrak{a}^r)$ be the set of all x for which $y \in \mathfrak{a}$ implies $xy = 0$ ($yx = 0$).

Lemma 7. $\mathfrak{a}^l$ is an l.i.—*Proof:* Obvious.

Lemma 8. $\mathfrak{a} \subset \mathfrak{b}$ implies $\mathfrak{a}^l \supset \mathfrak{b}^l$.—*Proof:* Obvious.

Lemma 9. $\mathfrak{a} \subset \mathfrak{a}^{lr}$.—*Proof:* Obvious.

Lemma 10. $\mathfrak{a}^l = \mathfrak{a}^{lrl}$.—*Proof:* Apply ^{l} to Lemma 9; then $\mathfrak{a}^l \supset \mathfrak{a}^{lrl}$ results by Lemma 8. Interchange r. and l. in Lemma 9 and then apply it to $\mathfrak{a}^l$ then $\mathfrak{a}^l \subset \mathfrak{a}^{lrl}$ results. Hence $\mathfrak{a}^l = \mathfrak{a}^{lrl}$.

Lemma 11. For every p.r.i. $\mathfrak{a}$ a p.l.i. $\mathfrak{b}$ with $\mathfrak{a} = \mathfrak{b}^r$ exists.—*Proof:* By Definition 4, $\mathfrak{a} = (e)_r$ for an ip. e. So $x \in \mathfrak{a}$ means $ex = x$, $(1 - e)x = 0$, that is $z(1 - e)x = 0$ for all $z \in \mathfrak{R}$. That is $yx = 0$ for all $y \in (1 - e)_l$, so $\mathfrak{a} = (1 - e)_l^r$. So $\mathfrak{b} = (1 - e)_l$ has the desired properties.

Lemma 12. If $\mathfrak{a}$ is a p.r.i., then $\mathfrak{a} = \mathfrak{a}^{lr}$.—*Proof:* By Lemma 11. $\mathfrak{a} = \mathfrak{b}^r$ for a p.l.i. $\mathfrak{b}$. Interchange r. and l. in Lemma 10, then $\mathfrak{a}^{lr} = \mathfrak{b}^{rlr} = \mathfrak{b}^r = \mathfrak{a}$ results.

Lemma 13. If $\mathfrak{a}$ is a p.r.i., then $\mathfrak{a}^l$ is a p.l.i.—*Proof:* By Lemma 11. $\mathfrak{a} = \mathfrak{b}^r$ for a p.l.i. $\mathfrak{b}$. Interchange r. and l. in Lemma 12. Then $\mathfrak{a}^l = \mathfrak{b}^{rl} = \mathfrak{b}$ results.—

By Lemma 13 (and its r.-l.-transposed) $\mathfrak{a} \longrightarrow \mathfrak{a}^r$ maps $R_{\mathfrak{R}}$ on part of $L_{\mathfrak{R}}$ and $\mathfrak{a} \longrightarrow \mathfrak{a}^l$ maps $L_{\mathfrak{R}}$ on part of $R_{\mathfrak{R}}$. By Lemma 12 (and its r.l.-transposed) these mappings are inverse to each other, hence one-to-one correspondences of $R_{\mathfrak{R}}$ and $L_{\mathfrak{R}}$. By Lemma 8 they are anti-monotonic. So we have proved:

Theorem 1. $\mathfrak{a} \longrightarrow \mathfrak{a}^l$ is a one-to-one mapping of $R_{\mathfrak{R}}$ on $L_{\mathfrak{R}}$, $\mathfrak{a} \longrightarrow \mathfrak{a}^r$ is a one-to-one mapping of $L_{\mathfrak{R}}$ on $R_{\mathfrak{R}}$. They are inverse to each other, and anti-monotonic.

4. We show now that $R_{\mathfrak{R}}$ and $L_{\mathfrak{R}}$ are lattices.[3]

Lemma 14. If $\mathfrak{a}$,$\mathfrak{b}$ are p.r.i., then two ip. e,f with $ef = fe = 0$ and $\mathfrak{a} \cup \mathfrak{b} = (e)_r \cup (f)_r$ exist.—*Proof:* By Definition 4, α), $\mathfrak{a} = (e_r)$, e ip. Besides $\mathfrak{b} = (b)_r$. Put $\mathfrak{b}_1 = ((1 - e)b)_r$. $\mathfrak{a} \cup \mathfrak{b}$ is the set of all $x + y$, $x \in \mathfrak{a}$, $y \in \mathfrak{b}$; that

is, the set of all $eu + bv, u, v \epsilon \mathfrak{R}$. $\mathfrak{a} \cup \mathfrak{b}_1$, is the set of all $x + y$, $x \epsilon \mathfrak{a}$, $y \epsilon \mathfrak{b}_1$, that is the set of all $eu' + (1 - e)bv = e(u' - bv) + bv, u', v \epsilon \mathfrak{R}$. The correspondence $u = u' - bv, u' = u + bv$ shows that $\mathfrak{a} \cup \mathfrak{b} = \mathfrak{a} \cup \mathfrak{b}_1$.

By Definition 4, α), $\mathfrak{b}_1 = (f_1)_r$, f_1 ip. As $f_1 \epsilon \mathfrak{b}_1 = ((1 - e)b)_r$, so $f_1 = (1 - e)bw$, hence $ef_1 = 0$. Now put $f = f_1(1 - e)$. Then $ff_1 = f_1(1 - e)f_1 = f_1(f_1 - ef_1) = f_1 f_1 = f_1$. Hence $f^2 = ff_1(1 - e) = f_1(1 - e) = f$, f ip. Since $f = f_1(1 - e) \epsilon (f_1)_r$, $f_1 = ff_1 \epsilon (f)_r$, so $(f)_r = (f_1)_r = \mathfrak{b}_1$. Hence $\mathfrak{a} \cup \mathfrak{b} = \mathfrak{a} \cup \mathfrak{b}_1 = (e)_r + (f)_r$.

Now $ef = ef_1(1 - \epsilon) = 0$, $fe = f_1(1 - e)e = 0$.

Lemma 15. If $\mathfrak{a}, \mathfrak{b}$ are p.r.i., then $\mathfrak{a} \cup \mathfrak{b}$ is a p.r.i. too.—*Proof:* Choose e, f as in Lemma 14. Then $e + f \epsilon (e)_r \cup (f)_r$, $(e + f)_r \subset (e)_r \cup (f)_r$. On the other hand $(e + f)e = e^2 + fe = e$, $(e + f)f = ef + f^2 = f$, hence e, $f \epsilon (e + f)_r$, $(e)_r$, $(f)_r \subset (e + f)_r$ and $(e)_r \cup (f)_r \subset (e + f)_r$. So $(e + f)_r = (e)_r \cup (f)_r = \mathfrak{a} \cup \mathfrak{b}$.

Lemma 16. If $\mathfrak{a}, \mathfrak{b}$ are p.r.i., then α) a minimum p.r.i. $\supset \mathfrak{a}, \mathfrak{b}$ exists, and it is $\mathfrak{a} \cup \mathfrak{b}$, β) a maximum p.r.i. $\subset \mathfrak{a}, \mathfrak{b}$ exists, and it is $\mathfrak{a} \cap \mathfrak{b}$.—*Proof:* Ad α): By Lemma 15. $\mathfrak{a} \cup \mathfrak{b}$ is a p.r.i., the other statements about $\mathfrak{a} \cup \mathfrak{b}$ are obvious. Ad β): Applying ^{r} to the p.l.i. $\mathfrak{a}^l, \mathfrak{b}^l$, and using Theorem 1, we see that the desired p.r.i. exists, and that it is equal to $(\mathfrak{a}^l \cup \mathfrak{b}^l)^r$. But this is equal to $\mathfrak{a}^{lr} \cap \mathfrak{b}^{lr}$ (obviously $(\mathfrak{c} \cup \mathfrak{d})^r = \mathfrak{c}^r \cap \mathfrak{d}^r$ for all l.i. $\mathfrak{c}, \mathfrak{d}$) that is to $\mathfrak{a} \cap \mathfrak{b}$ by Theorem 1 (or Lemma 12).—

Lemma 16, α), β), establish the lattice-character of $R_{\mathfrak{R}}$ with meet $\mathfrak{a} \cap \mathfrak{b}$ and join $\mathfrak{a} \cup \mathfrak{b}$. Clearly $(0) = (0)_r$ is the zero, and $\mathfrak{R} = (1)_r$ the unit. Definition 4, γ), secures for each p.r.i. $\mathfrak{a}$ an r.i. $\mathfrak{b}$ with $\mathfrak{a} \cap \mathfrak{b} = (0)$, $\mathfrak{a} \cup \mathfrak{b} = \mathfrak{R}$, and $\mathfrak{b}$ is a p.r.i. by Lemma 3. Hence $R_{\mathfrak{R}}$ is a "complemented" lattice (cf. G. Birkhoff,[4] p. 743, condition L 7, and footnote 3). Furthermore $R_{\mathfrak{R}}$ is a "modular" lattice (cf. G. Birkhoff[2], p. 445 where it is called a "B-lattice," and,[4] p. 743, condition L 5): (*) $\mathfrak{a} \subset \mathfrak{c}$ implies $(\mathfrak{a} \cup \mathfrak{b}) \cap \mathfrak{c} = \mathfrak{a} \cup (\mathfrak{b} \cap \mathfrak{c})$. Indeed: $(\mathfrak{a} \cup \mathfrak{b}) \cap \mathfrak{c} \supset \mathfrak{a} \cap \mathfrak{c} = \mathfrak{a}$ and $\supset \mathfrak{b} \cap \mathfrak{c}$; hence it is $\supset \mathfrak{a} \cup (\mathfrak{b} \cap \mathfrak{c})$. Next $x \epsilon (\mathfrak{a} \cup \mathfrak{b}) \cap \mathfrak{c}$ implies $x \epsilon \mathfrak{c}$ and $x \epsilon \mathfrak{a} \cup \mathfrak{b}$, that is $x = y + z$, $y \epsilon \mathfrak{a}$, $z \epsilon \mathfrak{b}$. Now $x \epsilon \mathfrak{c}$, $y \epsilon \mathfrak{a} \subset \mathfrak{c}$, so $z \epsilon \mathfrak{c}$, hence $z \epsilon \mathfrak{b} \cap \mathfrak{c}$, $x \epsilon \mathfrak{a} \cup (\mathfrak{b} \cap \mathfrak{c})$. Thus $(\mathfrak{a} \cup \mathfrak{b}) \cap \mathfrak{c} \subset \mathfrak{a} \cup (\mathfrak{b} \cap \mathfrak{c})$, proving (*).

Interchanging r. and l. establishes the same facts for $L_{\mathfrak{R}}$. Summing up, remembering Theorem 1:

Theorem 2. $R_{\mathfrak{R}}$ and $L_{\mathfrak{R}}$ are both *complemented, modular lattices*, with *meet* $\mathfrak{a} \cap \mathfrak{b}$, *join* $\mathfrak{a} \cup \mathfrak{b}$, *zero* (0), *unit* $\mathfrak{R}$. They are anti-isomorphic, the mappings of Theorem 1 being the anti-isomorphisms, interchanging $\subset$ and $\supset$, and consequently $\cup$ and $\cap$ too.

The Center. 5. Define as usual:

Definition 6. The center of $\mathfrak{R}$ is the set $\mathfrak{Z}$ of those $a \epsilon \mathfrak{R}$ which commute with every $x \epsilon \mathfrak{R}$: $ax = xa$.

$\mathfrak{Z}$ is a commutative ring with unit 1, and we prove:

Theorem 3. $\mathfrak{Z}$ is regular.—*Proof:* Consider an $a\epsilon\mathfrak{Z}$. As $\mathfrak{R}$ is regular, an $x\epsilon\mathfrak{R}$ with $axa = a$ exists. As a commutes with everything, so $a \cdot a^2x^3 \cdot a = axaxaxa = a$. Again $a^2x = xa^2 = axa = a$ commutes with everything, hence for every $z\epsilon\mathfrak{R}$, $x \cdot a^2z = xa^2 \cdot z = z \cdot xa^2 = a^2z \cdot x$,$a^2z$ commutes with x. Therefore x^3 commutes with it too, and so $a^2x^3 \cdot z = x^3 \cdot a^2z = a^2z \cdot x^3 = z \cdot a^2x^3$. Thus $x' = a^2x^3\epsilon\mathfrak{Z}$, and since $ax'a = a$, this establishes the regularity of $\mathfrak{Z}$.—

We discuss now those p.r.(l.)i. in $\mathfrak{R}$ which are generated by elements of $\mathfrak{Z}$. The same results hold for p.r.(l.)i. in $\mathfrak{Z}$ too: If we replace $\mathfrak{R}$ by $\mathfrak{Z}$ (which is permissible, since $\mathfrak{Z}$ is regular by Theorem 3), then the center remains $\mathfrak{Z}$.

Lemma 17. If $a\epsilon\mathfrak{Z}$ then $(a)_r = (a)_l$ and $(a)_r^l = (a)_l^r$. We denote this p.r. and p.l.i. by $(a)_*$ and $(a)_*{}^*$. *Proof:* As $ax = xa$ always, so $(a)_r = (a)_l$. $x\epsilon(a)_r^l$ means $xy = 0$ for all $y\epsilon(a)_r$, that is $xaz = 0$ for all z, that is $xa = 0$. $x\epsilon(a)_l^r$ means similarly $yx = 0$ for all $y\epsilon(a)_l$, that is $zax = 0$ for all z that is $ax = 0$. Now $xa = 0$ and $ax = 0$ are equivalent, hence $(a)_r^l = (a)_l^r$.

Lemma 18. α) A p.r.i. $\mathfrak{a}$ is at the same time an l.i. if and only if $\mathfrak{a} = (a)_*$ for an $a\epsilon\mathfrak{Z}$. We may even choose a as an ip. $e\epsilon\mathfrak{Z}$. β) If an l.i. $\mathfrak{a}$ is $= (e)_r$ for an ip. $e\epsilon\mathfrak{R}$, then this e is uniquely determined by $\mathfrak{a}$ and $e\epsilon\mathfrak{Z}$. Hence $\mathfrak{a} = (e)_*$.—*Proof:* Ad α): Sufficiency of $\mathfrak{a} = (a)_*$,$a\epsilon\mathfrak{Z}$: Obvious. Necessity of $\mathfrak{a} = (e)_*$, e an ip. $\epsilon\mathfrak{Z}$: As $\mathfrak{a}$ is a p.r.i., so $\mathfrak{a} = (e)_r$ for an ip. $e\epsilon\mathfrak{R}$ by Definition 4, α). Since $\mathfrak{a}$ is an l.i., so our β) gives $e\epsilon\mathfrak{Z}$, hence it suffices to prove β).

Ad β): Let $\mathfrak{a} = (e)_r$ be an l.i., e an ip. $\epsilon\mathfrak{R}$. Assume $x\epsilon\mathfrak{a}$. Then $x,e\epsilon\mathfrak{a}$, and as $\mathfrak{a}$ is an l.i., so $(x)_l$, $(e)_l$ and $(x)_l \cup (e)_l \subset \mathfrak{a}$. By Theorem 2 $(x)_l \cup (e)_l$ is a p.l.i., so it is $= (f)_l$, f ip., by Definition 4, β). So $f\epsilon(f)_l \subset \mathfrak{a} = (e)_r$, hence $ef = f$, and $e\epsilon(e)_l \subset (x)_l \cup (e)_l = (f)_l$, hence $ef = e$. Thus $f = e$, $x\epsilon(x)_l \subset (x)_l \cup (e)_l = (f)_l = (e)_l$. Since this holds for all $x\epsilon\mathfrak{a}$ so $\mathfrak{a} \subset (e)_l$. But we have proved $(e)_l \subset \mathfrak{a}$ already; therefore $\mathfrak{a} = (e)_l$.

Now $\mathfrak{a} = (e)_r = (e)_l$ gives for every x: $exe\epsilon(e)_r = \mathfrak{a} = (e)_l$, hence $exe = ex$, and $xe\epsilon(e)_l = \mathfrak{a} = (e)_r$, hence $exe = xe$. Thus $ex = xe$ which proves that $e\epsilon\mathfrak{Z}$. So $\mathfrak{a} = (e)_*$.

If $\mathfrak{a} = (g)_r$ for another ip. g, then we have: $g\epsilon(g)_r = \mathfrak{a} = (e)_l$, hence $ge = g$, and $e\epsilon(e)_l = \mathfrak{a} = (g)_r$, hence $ge = e$. So $g = e$.

Lemma 19. The only *reductions* of $\mathfrak{R}$ that is its decompositions into two r. and l.i. direct summands (cf. v.d.W. II, pp. 161–162), are these: $(^*_*)$ $\mathfrak{R} = (e)_* + (1 - e)_*$, e an ip. $\epsilon\mathfrak{Z}$.—*Proof:* In other words: The only pairs of inverse sets $\mathfrak{a}$,$\mathfrak{b}$(cf. Definition 2) which are both r. and l.i., are these: $\mathfrak{a} = (e)_*$,$\mathfrak{b} = (1 - e)_*$, e and ip. $\epsilon\mathfrak{Z}$. Sufficiency: Obvious. Necessity: $\mathfrak{a}$,$\mathfrak{b}$ are inverse r.i. so by Lemma 3, $\mathfrak{a} = (e)_r$, $\mathfrak{b} = (1 - e)_r$, e an ip. $\epsilon\mathfrak{R}$. As $\mathfrak{a}$ is l.i. too, so Lemma 18, β), gives $e\epsilon\mathfrak{Z}$. Hence $1 - e\epsilon\mathfrak{Z}$, $\mathfrak{a} = (e)_*$, $\mathfrak{b} = (1 - e)_*$.—

The *total reducibility* of $\mathfrak{R}$ would amount to this: For every r. and l.i. $\mathfrak{a}$ an inverse r. and l.i. $\mathfrak{b}$ can be found. (This is not the definition of v.d.W. II, pp. 161–162, which combines the above condition with the chain-condition). One can show by examples, however, that this is not always the case.

By Lemma 19 a necessary condition is that $\mathfrak{a}$ be even p.r.i. Conversely: If $\mathfrak{a}$ is p.r.i. (and also l.i.), then Lemma 18, α), gives $\mathfrak{a} = (e)_*$, e an ip. $\epsilon\mathfrak{Z}$, and so $\mathfrak{b} = (1 - e)_*$ is the desired inverse. So the p.r.i.- (or just as well the p.l.i.-) character of the r. and l.i. $\mathfrak{a}$ is necessary and sufficient for the existence of an inverse r. and l.i. $\mathfrak{b}$. Summing up:

Theorem 4. $\mathfrak{R}$ is *totally reducible* in the above restricted sense, and all *reductions* of $\mathfrak{R}$ are given by the ip. $e\epsilon\mathfrak{Z}$ in $(^*_*)$.—

We prove finally:

Theorem 5. $\mathfrak{R}$ is irreducible if and only if $\mathfrak{Z}$ is a division-algebra.—*Proof:* Owing to Lemma 19, the irreducibility of $\mathfrak{R}$ means that 0 and 1 are the only ip. $\epsilon\mathfrak{Z}$. Sufficiency of this condition: Let then a be $\epsilon\mathfrak{Z}$ and $\neq 0$. As $\mathfrak{Z}$ is regular, so $(a)_r = (e)_r$ in $\mathfrak{Z}$, e ip. $\epsilon\mathfrak{Z}$. $a \neq 0$ gives $e \neq 0$, so $e = 1$ $(a)_r = (1)_r = \mathfrak{Z}$ (in $\mathfrak{Z}$!), $1 = ax = xa$ for an $x\epsilon\mathfrak{Z}$. Thus a^{-1} exists in $\mathfrak{Z}$. Necessity: If $\mathfrak{Z}$ is a division-algebra, then for an ip. $e\epsilon\mathfrak{Z}$ we have $e(1 - e) = e - e^2 = 0$, so $e = 0$ or $1 - e = 0$, $e = 1$.

6. *Definition 7.* Let $Z_{\mathfrak{R}}$ be the intersection of $R_{\mathfrak{R}}$ and $L_{\mathfrak{R}}$.

By Lemma 18 $\mathfrak{a} = (e)_*$ establishes a one-to-one correspondence of all $\mathfrak{a}\epsilon Z_{\mathfrak{R}}$ and all ip. $e\epsilon\mathfrak{Z}$.

Lemma 20. If e is an ip. $\epsilon\mathfrak{R}$, then $(1 - e)xe = 0$ for all $x\epsilon\mathfrak{R}$ is equivalent to $e\epsilon\mathfrak{Z}$.—*Proof:* Sufficiency: If $e\epsilon\mathfrak{Z}$, then $(1 - e)xe = (1 - e)ex = 0$. Necessity: $(1 - e)xe = 0$, hence $exe = xe$, $xe\epsilon(e)_r$. As $(e)_r$ is an r.i., this implies $xey\epsilon(e)_r$, and as ey is the general element of $(e)_r$, so $u\epsilon(e)_r$ implies $xu\epsilon(e)_r$. That is: $(e)_r$ is an l.i. Hence $e\epsilon\mathfrak{Z}$ by Lemma 18, β).

Theorem 6. $Z_{\mathfrak{R}}$ is the set of those $\mathfrak{a}\epsilon R_{\mathfrak{R}}$ which possess a unique inverse $\mathfrak{b}\epsilon R_{\mathfrak{R}}$.—*Proof:* This means, owing to Lemmas 3 and 4: $\mathfrak{a}\epsilon Z_{\mathfrak{R}}$ if and only if $\mathfrak{a} = (e)_r$ for a unique ip. $e\epsilon\mathfrak{R}$. Sufficiency: $\mathfrak{a}\epsilon Z_{\mathfrak{R}}$ implies this by Lemma 18, β). Necessity: Assume $\mathfrak{a} = (e)_r$ for a unique ip. $e\epsilon\mathfrak{R}$. Put $e_1 = e + ex(1 - e)$. Then $e_1 e = e$, hence $e\epsilon(e_1)_r$ and $ee_1 = e_1$, hence $e_1\epsilon(e)_r$. So $\mathfrak{a} = (e)_r = (e_1)_r$. Besides ${e_1}^2 = e_1 e_1 = e_1.ee_1 = e_1 e.e_1 = ee_1 = e_1$, so e_1 ip. Hence $e_1 = e$, $ex(1 - e) = 0$. Now Lemma 20 (with $1 - e$ for e) gives $1 - e\epsilon\mathfrak{Z}$, $e\epsilon\mathfrak{Z}$.—

We have now characterized $Z_{\mathfrak{R}}$ in terms of the lattice $R_{\mathfrak{R}}$ (or just as well $L_{\mathfrak{R}}$) only. It would be easy to show that $Z_{\mathfrak{R}}$ is the set of all "neutral" elements in $R_{\mathfrak{R}}(L_{\mathfrak{R}})$ in the sense of O. Ore,[2] pp. 419–421.

Computation rules in the lattice $Z_{\mathfrak{R}}$:

Lemma 21. The unique inverse of $(e)_*$, e an ip. $\epsilon\mathfrak{Z}$, is $(e)_*{}^* = (1 - e)_*$ (cf. Lemma 17).—*Proof:* $(1 - e)_*$ is inverse to $(e)_*$ by Lemma 3, unique

by Theorem 6. $(e)_*^*$ is (cf. the proof of Lemma 17) the set of all x with $ex = 0$, that is $(1 - e)x = x$; hence it is equal to $(1 - e)_*$.

Lemma 22. If e,f are ip. $\epsilon\mathfrak{Z}$ then ef is too, and $(e)_* \cap (f)_* = (ef)_*$.—*Proof:* $ef\epsilon\mathfrak{Z}$ as $e,f\epsilon\mathfrak{Z}$, and $(ef)^2 = efef = eeff = ef$, so ef ip. $ef = fe$ is $\epsilon(e)_*$ and $(f)_*$, so $(ef)_* \subset (e)_*$, $(f)_*$ and $\subset (e)_* \cap (f)_*$. And $x\epsilon(e)_* \cap (f)_*$ gives $ex = fx = x$, so $efx = x$, $x\epsilon(ef)_*$. Hence $(e)_* \cap (f)_* \subset (ef)_*$. So $(e)_* \cap (f)_* = (ef)_*$.

Lemma 23. If e,f are ip. $\epsilon\mathfrak{Z}$ then $e + f - ef$ is too, and $(e)_* \cup (f)_* = (e + f - ef)_*$.—*Proof:* $e + f - ef = 1 - (1 - e)(1 - f)$ is an ip. $\epsilon\mathfrak{Z}$ along with e,f. By Theorem 2, $(e)_* \cup (f)_* = ((e)_*^* \cap (f)_*^*)^* = (^*$ coincides here with $^{r,l}) = (1 - (1 - e)(1 - f))_* = (e + f - ef)_*$.—

It is now easy to verify that $Z_{\mathfrak{R}}$ is a distributive lattice or a *Boolean algebra*, that is, that for all ip. $e,f,g\epsilon\mathfrak{Z}$ the distributive law $((e)_* \cup (f)_*) \cap (g)_* = ((e)_* \cap (g)_*) \cup ((f)_* \cup (g)_*)$ holds. Indeed: By Lemmas 22–23 both sides are equal to $(eg + fg - efg)_*$. Se we see:

Theorem 7. $Z_{\mathfrak{R}}$ is a *complemented Boolean algebra.*

Remarks. 7. *Definition 8.* Given an ip. e let $\mathfrak{R}(e)$ be the set of all $x\epsilon\mathfrak{R}$ with $ex = xe = x$. Given an $n = 1,2,\ldots$ let $\mathfrak{R}_n$ be the set of all nth order matrices (x_{ij}) $i,j = 1,\ldots, n$ of elements $x_{ij}\epsilon\mathfrak{R}$.[5] Clearly $\mathfrak{R}(e)$ is a ring with unit e, and $\mathfrak{R}_n$ is a ring with unit $1_n = (\delta_{ij})$ $\delta_{ij} = 1$ for $i = j$ and $= 0$ for $i \neq j$.

$\mathfrak{R}(e)$ is regular: If $a\epsilon\mathfrak{R}(e)$ then an $x\epsilon\mathfrak{R}$ with $axa = a$ exists. Now $e\cdot exe\cdot = exe = exe\cdot e$, hence $x' = exe\epsilon\mathfrak{R}(e)$, and $ax'a = a\cdot exe\cdot a = ae\cdot xea = axa = a$.

$\mathfrak{R}_n$ is regular too. We omit the proof, which is best based on a discussion of systems of linear equations in $\mathfrak{R}$ leading to a verification of Definition 4, γ, for $\mathfrak{R}_n$.

[1] B. L. van der Waerden, "Moderne Algebra," Vols. I and II. J. Springer, 1930, to be quoted as v.d.W. I and II, respectively.

[2] F. Klein, *Math. Ann.*, **105**, 308–323 (1931); G. Birkhoff, *Proc. Cambridge Phil. Soc.*, **29**, 441–464 (1933); O. Ore, *Ann. Math.*, **36**, 406–437 (1935). We shall use the terminology of G. Birkhoff's paper.

[3] While all right ideals necessarily form a lattice, this is not so in general for the principal ones: The fact that in rings of algebraic integers α, $\mathfrak{b}$ may be principal ideals while $\mathfrak{a} \cup \mathfrak{b}$ (their "greatest common divisor") is not one, is the origin of Kummer's theory of ideals!

[4] G. Birkhoff, *Ann. Math.*, **37**, 743–748 (1936).

[5] With the usual definitions of $+$ and $\cdot$: $(x_{ij}) + (y_{ij}) = (x_{ij} + y_{ij})$, $(x_{ij}) \cdot (y_{ij}) = \sum_{k=1}^{n} x_{ik}y_{kj}$.

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